HEAT AND MASS TRANSFER

Solved Problems

By

Mr. P. Raveendiran

Asst. Professor, Mechanical

Heat and mass Transfer

Unit I

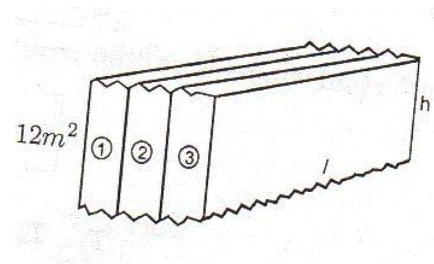
November 2008

1. Calculate the rate of heat loss through the vertical walls of a boiler furnace of size 4 m by 3 m by 3 m high. The walls are constructed from an inner fire brick wall 25 cm thick of thermal conductivity 0.4 W/mK, a layer of ceramic blanket insulation of thermal conductivity 0.2 W/mK and 8 cm thick, and a steel protective layer of thermal conductivity 55 W/mK and 2 mm thick. The inside temperature of the fire brick layer was measured at 600^o C and the temperature of the outside of the insulation 60^0 C. Also find the interface temperature of layers.

Given:

Composite Wall

l= 4m b= 3m h= 3m



Area of rectangular wall lb = 4x3 = 12m^2

L_1 = 25 cm, k_1 = **0.4 W/mK** } Fire brick

L_2 =0.002m, k_2 = **54 W/mK** } Steel

L_3 = 0.08 m, k_1 = **0.2 W/mK** } insulation

$T_1 = 600^0$ C

$T_2 = 60^0$ C

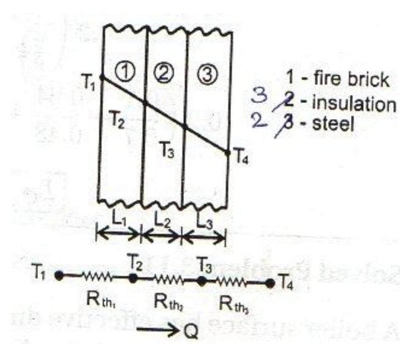
Find

(i) Q (ii) (T_3–T_4)

Solution

We know that,

$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$



Here

(ΔT) overall = $T_1 - T_4$

And $\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$$R_{th1} = \frac{L_1}{k_1 A} = \frac{0.25}{0.4x12} = 0.0521 K/W$$

$$R_{th2} = \frac{L_2}{k_2 A} = \frac{0.08}{0.2x12} = 0.0333 K/W$$

$$R_{th3} = \frac{L_3}{k_3 A} = \frac{0.002}{54x12} = 0.0000031 K/W$$

$$Q = \frac{T_1 - T_4}{R_{th1}+R_{th2}+R_{th3}}$$

$$= \frac{600 - 60}{0.0521 + 0.0000031 + 0.0333}$$

Q = 6320.96 W

(i) To find temperature drop across the steel layer (T_2 - T_3)

$$Q = \frac{T_2 - T_3}{R_{th3}}$$

T_3- T_4 = Q× R_{th2}

= 6320.96× 0.0000031

T_3- T_4 = 0.0196 K .

2. A spherical container of negligible thickness holding a hot fluid at 140^0 and having an outer diameter of 0.4 m is insulated with three layers of each 50 mm thick insulation of k_1 = 0.02: k_2 = 0.06 and k_3 = 0.16 W/mK. (Starting from inside). The outside surface temperature is 30^{0}C. Determine (i) the heat loss, and (ii) Interface temperatures of insulating layers.

Given:

OD = 0.4 m

r_1 = 0.2 m

r_2 = r_1 + thickness of 1st insulation

= 0.2+0.05

r_2 = 0.25m

r_3 = r_2 + thickness of 2nd insulation

= 0.25+0.05

r_3 = 0.3m

r_4 = r_3 + thickness of 3rd insulation

= 0.3+0.05

r_4 = 0.35m

T_{hf} = 140^o C, T_{cf} = 30^o C,

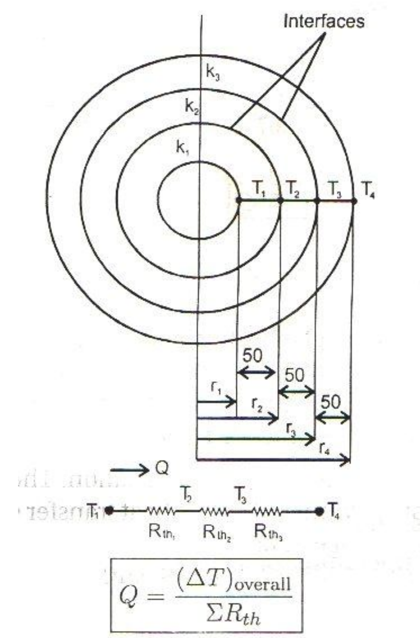
k_1 = 0.02 W/mK

k_2 = 0.06 W/mK

k_3 = 0.16 W/mK.

Find (i) Q (ii) T_2, T_3

Solution



$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

$\Delta T = T_{hf} - T_{cf}$

$\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$R_{th1} = \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} = \frac{(0.25-0.20)}{4\pi\, x 0.02 x 0.25 x 0.2}$ =3.978° C/W

$R_{th2} = \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} = \frac{(0.30-0.25)}{4\pi\, x 0.06 x 0.3 x 0.25}$ =0.8842° C/W

$R_{th1} = \frac{r_4 - r_3}{4\pi k_3 r_4 r_3} = \frac{(0.35-0.30)}{4\pi\, x 0.16 x 0.35 x 0.30}$ =0.23684° C/W

$$Q = \frac{140 - 30}{0.0796 + 0.8842 + 0.23684}$$

Q = 21.57 W

To find interface temperature (T_2 , T_3)

$Q = \frac{T_2 - T_3}{R_{th1}}$

$T_2 = T_1 - [Q \text{ x } R_{th1}]$

= 140 – [91.62×0.0796]

$T_2 = 54.17^0C$

$Q = \frac{T_2 - T_3}{R_{th1}}$

$T_3 = T_2 - [Q \times R_{th2}]$

= 132.71- [91.62×0.8842]

$T_3 = 35.09^o$ C

3. May 2008

A steel tube with 5 cm ID, 7.6 cm OD and k=15W/m ^{o} C is covered with an insulative covering of thickness 2 cm and k 0.2 W/m ^{o}C. A hot gas at 330^o C with h = 400 W/m^{2o}C flows inside the tube. The outer surface of the insulation is exposed to cooler air at 30^{o}C with h = 60 W/m^{2o}C. Calculate the heat loss from the tube to the air for 10 m of the tube and the temperature drops resulting from the thermal resistances of the hot gas flow, the steel tube, the insulation layer and the outside air.

Given:

Inner diameter of steel, d_1 = 5 cm =0.05 m

Inner radius,r_1 = 0.025m

Outer diameter of steel, d_2 = 7.6 cm = 0.076m

Outer radius,r_2 = 0.025m

Radius, $r_3 = r_2$ + thickness of insulation

= 0.038+0.02 m

$r_3 = 0.058$ m

Thermal conductivity of steel, $k_1 = 15 W/m\,^oC$

Thermal conductivity of insulation, $k_2 = 0.2 W/m\,^oC$

Hot gas temperature, $T_{hf} = 330^o C + 273 = 603 K$

Heat transfer co-efficient at innear side, $h_{hf} = 400 W/m^{2o}C$

Ambient air temperature, $T_{cf} = 30^oC + 273 = 303 K$

Heat transfer co-efficient at outer side $h_{cf} = 60 W/m^{2o}C$.

Length, L = 10 m

To find:

(i) Heat loss (Q)

(ii) Temperature drops $(T_{hf} - T_1)$, $(T_1 - T_2)$, $(T_2 - T_3)$, $(T_3 - T_{cf})$,

Solution:

Heat flow $Q = \frac{\Delta T_{overall}}{\Sigma R_{th}}$

Where

$\Delta T_{overall} = T_{hf} - T_{cf}$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_{hf}r_1} + \frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{k_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{k_3}\ln\left[\frac{r_4}{r_3}\right] + \frac{1}{h_{cf}r_4}\right]$$

$$Q = \frac{T_{hf} - T_{cf}}{\frac{1}{2\pi L}\left[\frac{1}{h_{hf}r_1} + \frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{k_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_{cf}r_3}\right]}$$

$$Q = \frac{603 - 303}{\frac{1}{2\pi \times 10}\left[\frac{1}{400 \times 0.025} + \frac{1}{15}\ln\left[\frac{0.038}{0.025}\right] + \frac{1}{0.2}\ln\left[\frac{0.058}{0.038}\right] + \frac{1}{60 \times 0.058}\right]}$$

Q = 7451.72 W

We know that,

$$Q = \frac{T_{hf} - T_1}{R_{th\ conv.}}$$

$$= \frac{T_{hf} - T_1}{\frac{1}{2\pi L} \times \frac{1}{h_{hf}r_1}}$$

$$7451.72 = \frac{T_{hf} - T_1}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{400 \times 0.025}}$$

$$T_{hf} - T_1 = 11.859K$$

$$Q = \frac{T_1 - T_2}{R_{th1}}$$

$$= \frac{T_1 - T_2}{\frac{1}{2\pi L} \times \left[\frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right]\right]}$$

$$7451.72 = \frac{T_1 - T_2}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{15} \ln \left[\frac{0.038}{0.025}\right]}$$

$$T_1 - T_2 = 3.310\ K$$

$$Q = \frac{T_2 - T_3}{R_{th2}}$$

$$= \frac{T_2 - T_3}{\frac{1}{2\pi L} \times \left[\frac{1}{k_2} \ln\left[\frac{r_3}{r_2}\right]\right]}$$

$$7451.72 = \frac{T_2 - T_3}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{0.2} \ln \left[\frac{0.058}{0.038}\right]}$$

$$T_2 - T_3 = 250.75\ K$$

$$Q = \frac{T_3 - T_{cf}}{R_{th\ conv.}}$$

$$= \frac{T_3 - T_{cf}}{\frac{1}{2\pi L} \times \frac{1}{h_{cf} r_3}}$$

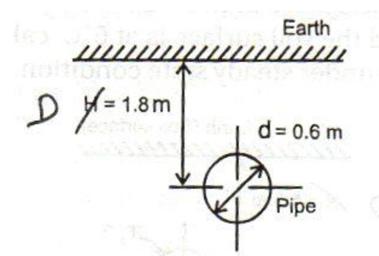
$$7451.72 = \frac{T_3 - T_{cf}}{\frac{1}{2 \times \pi \times 10} \times \left[\frac{1}{60 \times 0.058}\right]}$$

$$T_3 - T_{cf} = 34.07K$$

Nov 2009

4. A long pipe of 0.6 m outside diameter is buried in earth with axis at a depth of 1.8 m. the surface temperature of pipe and earth are 95^0 C and 25^0 C respectively. Calculate the heat loss from the pipe per unit length. The conductivity of earth is 0.51W/mK.

Given



$r = \frac{0.6}{2} = 0.3$ m

L = 1 m

$T_p = 95^o$ C

$T_e = 25^o$ C

D = 1.8 m

k = 0.51W/mK

Find

Heat loss from the pipe (Q/L)

Solution

We know that

$$\frac{Q}{L} = k.S(T_p - T_e)$$

Where S = Conduction shape factor =

$$\frac{2\pi L}{\ln\left(\frac{2D}{r}\right)}$$

$$= \frac{2\pi x\ 1}{\ln\left(\frac{2x\ 1.8}{0.3}\right)}$$

S = 2.528m

$$\frac{Q}{L} = 0.51x2.528(95 - 25)$$

$$\frac{Q}{L} = 90.25W/m$$

Nov.2010

5. A steam pipe of 10 cm ID and 11 cm OD is covered with an insulating substance k = 1 W/mK. The steam temperature is 200^0 C and ambient temperature is 20^0 C. If the convective heat transfer coefficient between insulating surface and air is 8 W/m^2K, find the critical radius of insulation for this value of r_c. Calculate the heat loss per m of pipe and the outer surface temperature. Neglect the resistance of the pipe material.

Given:

$$r_{i=}\frac{ID}{2} = \frac{10}{2} = 5\ cm = 0.05m$$

$$r_{0=}\frac{OD}{2} = \frac{11}{2} = 5.5\ cm = 0.055m$$

k =1 W/mK

$T_i = 200^oC$ $T_\infty = 20^o$ C

$h_0 = 8\ W/m^2K$

Find

(i) r_c

(ii) If $r_c = r_o$ then Q/L

(iii) T_o

Solution

To find critical radius of insulation (r_c)

$$r_{0=}\frac{k}{h_0} = \frac{1}{8} = 0.125m$$

When $r_c = r_o$

Kpipe, h_{hf} not given

$$\frac{Q}{L} = \frac{2\pi(T_0 - T_\infty)}{\frac{\ln\left(\frac{r_c}{r_o}\right)}{k} + \frac{1}{h_o r_o}}$$

$$= \frac{2\pi(200-20)}{\frac{\ln\left(\frac{0.125}{0.050}\right)}{1} + \frac{1}{8\,x\,0.125}}$$

$$\frac{Q}{L} = 621\,W/m$$

To Find T_o

$$\frac{Q}{L} = \frac{T_0 - T_\infty}{R_{thconv}}$$

$$T_0 = T_\infty + \frac{Q}{L}(R_{thconv})$$

$$= 20 + 621 \times \left(\frac{1}{8 \times 2\pi \times 0.125}\right)$$

$T_0 = 118.72^0C$

November 2011.

6. The temperature at the inner and outer surfaces of a boiler wall made of 20 mm thick steel and covered with an insulating material of 5 mm thickness are 300^0 C and 50^0 C respectively. If the thermal conductivities of steel and insulating material are $58W/m^0C$ and $0.116\ W/m^0C$ respectively, determine the rate of flow through the boiler wall.

$L1 = 20 \times 10^{-3}$ m

$k_1 = 58\ W/m^0C$

$L_2 = 5 \times 10^{-3}$ m

$k_2 = 0.116\ W/m^0C$

$T_1 = 300^0$ C

$T_2 = 50^0$ C

Find

(i) Q

Solution

$$Q = \frac{(\Delta T)overall}{\Sigma Rth} = \frac{T_1 - T_3}{R_{th1} - R_{th2}}$$

$$R_{th1} = \frac{L1}{k1A} = \frac{0.20 \times 10^{-3}}{58 \times 1} = 3.45 \text{ X } 10^{-4}\ {}^0C/W$$

$$R_{th2} = \frac{L2}{k2A} = \frac{5 \times 10^{-3}}{0.116 \times 1} = 0.043\ {}^0C/W$$

$$Q = \frac{300 - 50}{3.45 \text{ X } 10-4 + 0.043} = 5767.8 \text{ W}$$

Q = 5767.8 W

7. A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 200° C. Thermal conductivity of material is 0.3 kJ /mh°C.

Given

d_1 =1.2 m

r_1 = 0.6 m

r_2 = r_1 + thick

= 0.6 + 0.1

r_2 = 0.7 m

ΔT =200°C

K = 0.3 kJ /mhr °C = 0.0833 W/m° C

Find

Q

Solution:

$$Q = \frac{\Delta T}{R_{th}} = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th} = \frac{r_2 - r_1}{4\pi r_2 r_1} = \frac{(0.7-0.6)}{4\pi \times 0.0833 \times 0.6 \times 0.7} = 0.2275\ K/W$$

$$Q = \frac{\Delta T}{R_{th}} = \frac{200}{0.2275} = 879.132 W$$

November 2011 (old regulation)

8. A steel pipe (K = 45.0 W/m.K) having a 0.05m O.D is covered with a 0.042 m thick layer of magnesia (K = 0.07W/m.K) which in turn covered with a 0.024 m layer of fiberglass insulation (K = 0.048 W/m.K). The pipe wall outside temperature is 370 K and the outer surface temperature of the fiberglass is 305K. What is the interfacial temperature between the magnesia and fiberglass? Also calculate the steady state heat transfer.

Given:

OD = 0.05 m

d_1= 0.05 m

r_1 = 0.025 m

k_1 = 45 W/mK

r_2 = r_1 + thick of insulation 1

r_2 = 0.025+0.042

r_2 = 0.067 m

k_2 = 0.07 W/mK

$k_3 = 0.048$ W/mK

$r_3 = r_2$ + thick of insulation 2

$= 0.067+0.024$

$r_3 = 0.091$ m

$T_1 = 370$ K

$T_3 = 305$ K

To find

(i) T_2

(ii) Q

Solution

Here thickness of pipe is not given; neglect the thermal resistance of pipe.

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

$$(\Delta T)overall = T_1 - T_3 = 370 - 305 = 65\,K$$

$\Sigma R_{th} = R_{th1} + R_{th2}$

$$R_{th1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{2L}} = \frac{\ln\left(\frac{0.067}{0.025}\right)}{2\pi \times 0.07 \times 1} = 2.2414 \text{ K/W}$$

$$R_{th2} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{3L}} = \frac{\ln\left(\frac{0.091}{0.067}\right)}{2\pi \times 0.48 \times 1} = 1.0152 \text{ K/W}$$

$$Q = \frac{65}{2.2414+1.0152} = 19.959 \text{ W/m}$$

To find T_2

$$Q = \frac{T_1 - T_2}{R_{th1}}$$

$T_2 = T_1 - [Q \text{ x } R_{th1}]$

$= 370 - [19.959 \text{ x } 2.2414]$

$T_3 = 325.26$K

9. A motor body is 360 mm in diameter (outside) and 240 mm long. Its surface temperature should not exceed 55 ^oC when dissipating 340W. Longitudinal fins of 15 mm thickness and 40 mm height are proposed. The convection coefficient is 40W/m^2 ^oC. determine the number of fins required. Atmospheric temperature is 30^oC. thermal conductivity = 40 W/m^oC.

Given:

$D = 360 \times 10^{-3}$ m

$L = 240 \times 10^{-3}$ m

$T_b = 55^oC$

$Q_{generating} = 340W$

Longitudinal fin

$t_{fin} = 15 \times 10^{-3}$ m

$h_{fin} = 40 \times 10^{-3}$ m

$h = 40 W/m^2 {}^oC$

$k = 40 W/m {}^oC$.

$T_\infty = 30 {}^oC$

To find:

No of fins required (N)

Solution:

Here length (or) height of fin is given. It is short fin(assume end insulated)

$$N = \frac{Q_{gen}}{Q_{per\ fin}}$$

From HMT Data book,

$$Q = \sqrt{hPkA}\ (T_b - t_\infty). \tan h(mL)$$

$$m = \sqrt{\frac{hP}{kA}}\ m^{-1}$$

Perimeter (P) = 2L = 2 x 0.24 = 0.48 m

(for longitudinal fin fitted on the cylinder)

Area (A) = Lt = 0.24 x 0.015

A = 0.0036m^2

$$m = \sqrt{\frac{40 \times 0.48}{40 \times 0.0036}} = 11.55\ m^{-1}$$

$$Q_{fin} = \sqrt{40 \times 0.48 \times 40 \times 0.0036}\ (55 - 30). \tan h(11.55 \times 0.04)$$

$Q_{fin} = 4.718$ W

$$N = \frac{340}{4.718} = 72.06 = 72\ fins.$$

May 2012

10. A mild steel tank of wall thickness 10 mm contains water at 90^o C. The thermal conductivity of mild steel is 50 W/m^{o}C , and the heat transfer coefficient for inside and outside of the tank area are 2800 and 11 W/m^2 ^{o}C, respectively. If the atmospheric temperature is 20^{o}C , calculate

(i) The rate of heat loss per m^2 of the tank surface area.

(ii) The temperature of the outside surface tank.

Given

$L = 10 \times 10^{-3}$ m

$T_{hf} = 90\ ^oC$

$k = 50$ W/m ^oC

$h_{hf} = 2800$ W/m^2 ^oC

$h_{cf} = 11$ W/m^2 ^oC

$T_{cf} = 20^o$ C

To find

(i) Q/m^2

(ii) T_2

Solution

$$Q = \frac{(\Delta T) overall}{\Sigma Rth}$$

Here $(\Delta T)_{overall} = T_{hf} - T_{cf} = 90 - 20 = 70^oC$

$$\sum R_{th} = R_{th_{conv_{hf}}} + R_{th1} + R_{th_{conv_{cf}}}$$

$$R_{th_{conv_{hf}}} = \frac{1}{h_{hf}.A} = \frac{1}{2800\times1} 0.00036\ K/W$$

$$R_{th} = \frac{L}{kA} = \frac{10 \times 10^{-3}}{50\times 1} = 0.0002\ K/W$$

$$R_{th_{conv_{cf}}} = \frac{1}{h_{cf}.A} = \frac{1}{11\times 1} 0.09091\ K/W$$

$$Q = \frac{70}{0.091469} = 765.29\ W/m^2$$

To find T_2

$$Q = \frac{T_{hf} - T_2}{R_{conv_{hf}+R_{th\,1}}}$$

$$T_{2=} T_{hf} - \left[Q \times R_{conv_{hf}+R_{th\,1}}\right]$$

$= 90 - [765\text{x } 0.00056]$

$T_2 = 89.57\ ^0C$

11. A 15 cm outer diameter steam pipe is covered with 5 cm high temperature insulation (k = 0.85 W/m °C) and 4 cm of low temperature (k = 0.72 W/m° C). The steam is at 500 °C and ambient air is at 40 °C. Neglecting thermal resistance of steam and air sides and metal wall calculate the heat loss from 100 m length of the pipe. Also find temperature drop across the insulation.

Given

d_1	=	15 cm
r_1	=	7.5×10^{-2} m
r_2	=	r_1 + thick of high temperature insulation
r_2	=	$7.5 + 5 = 12.5 \times 10^{-2}$ m
r_3	=	r_2 + thick of low temperature insulation
r_3	=	$12.5 + 4 = 16.5 \times 10^{-2}$ m
k_{ins1}	=	0.85 w/m° C
k_{ins2}	=	0.72 w/m° C
T_{hf}	=	500 ° C
T_{cf}	=	40 ° C

To find

(i) Q if L = 1000mm = 1 m

Solution:

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

$$\Delta T = T_1 - T_3$$

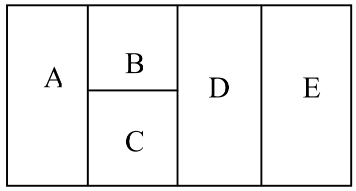
$$\Sigma R_{th} = R_{th1} + R_{th2}$$

$$R_{th1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{1L}} = \frac{\ln\left(\frac{0.125}{0.075}\right)}{2\pi \times 0.85 \times 1} = 0.09564 \text{ K/W or } {}^{\circ}\text{C/W}$$

$$R_{th2} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{2L}} = \frac{\ln\left(\frac{0.165}{0.125}\right)}{2\pi \times 0.72 \times 1} = 0.06137 \text{ K/W or } {}^{\circ}\text{C/W}$$

$$Q = \frac{500-40}{0.09564+0.06137} = 2929.75 \text{W/m}$$

12. Determine the heat transfer through the composite wall shown in the figure below. Take the conductives of A, B, C, D & E as 50, 10, 6.67, 20& 30 W/mK respectively and assume one dimensional heat transfer. Take of area of A =D= E = 1m^2 and B=C=0.5 m^2. Temperature entering at wall A is 800 ^{o} C and leaving at wall E is 100^o C.



Given:

$T_i = 800^o C$

$T_o = 100^o C$

$k_A = 50$ W/mK

$k_B = 10$ W/mK

$k_c = 6.67$ W/mK

$k_D = 20$ W/mK

$k_E = 30$ W/mK

$A_A = A_D = A_E = 1 m^2$

$A_B = A_C = 0.5 m^2$

Find

(i) Q

Solution

$$Q = \frac{(\Delta T) overall}{\Sigma Rth}$$

$$R_{th1} = R_{thA} = \frac{L_A}{k_A\ A}$$

Parallel $\frac{1}{R_{th2}} = \frac{1}{R_{thB}} + \frac{1}{R_{thC}} = \frac{R_{thB}+R_{thC}}{R_{thB}R_{thC}}$

$$R_{th2} = \frac{R_{thB}R_{thC}}{R_{thB} + R_{thC}}$$

$$R_{thB} = \frac{L_B}{k_B A_B}$$

$$R_{thC} = \frac{L_C}{k_C A_C}$$

$$R_{th4} = R_{thE} = \frac{L_E}{k_E A_E}$$

$$R_{th3} = R_{thD} = \frac{L_D}{k_D A_D}$$

$$R_{th1} = R_{thA} = \frac{1}{50 \times 1} = 0.02\ K/W$$

$$R_{thB} = \frac{1}{10 \times 0.5} = 0.2\ K/W$$

$$R_{thC} = \frac{1}{6.67 \times 0.5} = 0.2969\ K/W$$

$$R_{th2} = \frac{R_{thB}R_{thC}}{R_{thB} + R_{thC}} = \frac{0.2 \times 0.299}{0.2 + 0.299} = \frac{0.0598}{0.499}$$

$$R_{th2} = 0.1198\ K/W$$

$$R_{th3} = R_{thD} = \frac{L_D}{K_D A_D} = \frac{1}{20 \times 1} = 0.05\ K/W$$

$$R_{th4} = R_{thE} = \frac{L_E}{K_E A_E} = \frac{1}{30 \times 1} = 0.0333\ K/W$$

$$Q = \frac{T_i - T_o}{\sum R_{th}} = \frac{800 - 100}{0.02 + 0.1198 + 0.05 + 0.0333} = 3137.61W$$

$$Q = 3137.61W$$

13. A long carbon steel rod of length 40 cm and diameter 10 mm (k = 40 w/mK) is placed in such that one of its end is 400^o C and the ambient temperature is 30^o C. the flim co-efficient is 10 w/m^2K. Determine

(i) Temperature at the mid length of the fin.

(ii) Fin efficiency

(iii) Heat transfer rate from the fin

(iv) Fin effectiveness

Given:

$l = 40x10^{-2}$ m

d = 10 x 10^{-3} m

k = 40 W/mK

$T_b = 400^o$ C

$T_\infty = 30\ ^o$ C

H = 10 w/m^2K

To find

(i) T , x = L/2

(ii) η_{fin}

(iii) Q_{fin}

Solution

It is a short fin end is insulated

From H.M.T Data book

$$Q = \sqrt{hPkA}\ (T_b - T_\infty).\tan h(mL)$$

$$m = \sqrt{\frac{hP}{kA}}\ m^{-1}$$

Perimeter = πd = π x 10 x 10^{-3} = 0.0314 m

$$Area = \frac{\pi}{4}d^2 = \frac{\pi}{4}(10 \times 10^{-3})^2 = 0.0000785\ m^2$$

$$m = \sqrt{\frac{10 \times 0.0314}{40 \times 0.0000785}} = 10\ m^{-1}$$

$$Q = \sqrt{10 \times 0.0314 \times 40 \times 0.0000785}\ (400 - 30).\tan h(10 \times 40 \times 10^{-2})$$

Q = 0.115 W

From H.M.T Data book

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos hm\ (L - x)}{\cos h\ (mL)}$$

$$\frac{T - 30}{400 - 30} = \frac{\cos h\ 10\ (0.4 - 0.2)}{\cos h\ (10 \times 0.4)}$$

$$\frac{T - 30}{400 - 30} = \frac{3.762}{27.308}$$

$$\frac{T - 30}{370} = 0.13776$$

T = 50.97 + 30

T = 80.97 ^{o}C

14. A wall furnace is made up of inside layer of silica brick 120 mm thick covered with a layer of magnesite brick 240 mm thick. The temperatures at the inside surface of silica brick wall and outside the surface of magnesite brick wall are 725^{o}C and 110^{o}C respectively. The contact thermal resistance between the two walls at the interface is 0.0035^{o}C/w per unit wall area. If thermal conductivities of silica and magnesite bricks are 1.7 W/m^{o}C and 5.8 W/m^{o}C, calculate the rate of heat loss per unit area of walls.

Given:

L_1 = 120 x 10^{-3} m

k_1 = 1.7 W/m^{0}C

L_2 = 240 x 10^{-3} m

k_2 = 5.8 W/m^{0}C

T_1 = 725 ^{0} C

T_4 = 110^{0} C

$(R_{th})_{contact} = 0.0035\ {}^{o}C/W$

Area = 1 m^{2}

Find

(i) Q

Solution

$$Q = \frac{(\Delta T)overall}{\Sigma Rth} = \frac{T_1 - T_4}{\text{Rth1} + (R_{th})_{cont} + \text{Rth2}}$$

Here $T_1 - T_4 = 725 - 110 = 615^o$ C

$$R_{th1} = \frac{L1}{k1A} = \frac{120 \text{ x } 10^{-3}}{1.7 \times 1} = 0.0706^0 \text{ C /W}$$

$$R_{th2} = \frac{L2}{k2A} = \frac{240 \text{x } 10^{-3}}{5.8 \times 1} = 0.0414\ ^0 \text{C /W}$$

$$Q = \frac{615}{0.0706 + 0.0035 + 0.0414} = 5324.67 \text{ W/m}^2$$

Q = 5324.67 W/m

15. A furnace walls made up of three layers , one of fire brick, one of insulating brick and one of red brick. The inner and outer surfaces are at 870^o C and 40^o C respectively. The respective co- efficient of thermal conduciveness of the layer are 1.0, 0.12 and 0.75 W/mK and thicknesses are 22 cm, 7.5, and 11 cm. assuming close bonding of the layer at their interfaces, find the rate of heat loss per sq.meter per hour and the interface temperatures.

Given

Composite wall (without convection)

$L_1 = 22 \text{ x}10^{-2}$ m

$k_1 = 1$ W/mK

$L_2 = 7.5 \text{ x}10^{-2}$ m

$k_2 = 0.12$ W/mK

$L_3 = 11\text{x}10^{-2}$ m

$k_3 = 0.75$ W/mK

$T_1 = 870^o$ C

$T_4 = 40^o$ C

Find

(i) Q / hr (ii) T_2, T_3

Solution

We know that,

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

(ΔT) overall = $T_1 - T_4$

= 870 – 40

$= 830\,^oC$

And $\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

(assume $A = 1\ m^2$)

$R_{th1} = \frac{L1}{k1A} = \frac{22\ x10-2}{1\times1} = 22\ x10^{-2}$ K/W

$R_{th2} = \frac{L2}{k2A} = \frac{7.5\ x10-2}{0.12\times1} = 0.625$ K/W

$R_{th3} = \frac{L3}{k3A} = \frac{11x10-2}{0.75\times1} = 0.1467$ K/W

$$Q = \frac{T1 - T4}{Rth1 + Rth2 + Rth3}$$

$$= \frac{870 - 40}{0.9917}$$

$Q = 836.95\ \ W/m^2$

$Q = 3.01X\ 10^5$ J/h

Nov 2010

16. A 12 cm diameter long bar initially at a uniform temperature of $40^{o}C$ is placed in a medium at $650^{o}C$ with a convective co efficient of 22 W/m^2K calculate the time required for the bar to reach$255^{0}C$. Take k = 20W/mK, ρ = 580 kg/m^3 and c = 1050 J/kg K.

Given : Unsteady state

D = 12 cm = 0.12 m

R = 0.06 m

$T_o = 40 + 273 = 313$ K

$T_\infty = 650 + 273 = 923$ K

T = 255 + 273 =528 K

h = 22 W/m^2K

k = 20 W/mK

ρ = 580 Kg/m^3

c = 1050 J/kg K

Find:

Time required to reach $255^{o}C$ (τ)

Solution

Characteristic length for cylinder = $L_c = \frac{R}{2}$

$$L_c = \frac{0.06}{2} = 0.03 \text{ m}$$

We know that

$$B_i = \frac{hL_c}{k} = \frac{22 \times 0.03}{20}$$

$$B_i = 0.033 < 0.1$$

Biot number is less than 0.1. Hence this is lumped heat analysis type problem.

For lumped heat parameter, from HMT data book.

$$\frac{T-T_\infty}{T_o-T_\infty} = e^{\left[-\frac{h\,A}{c\,V\,\rho}\times\tau\right]}$$

We know that

$$L_c = \frac{V}{A}$$

$$\frac{T-T_\infty}{T_o-T_\infty} = e^{\left[\frac{-h}{c\,L_c\rho}\times\tau\right]}$$

$$\frac{528-923}{313-923} = e^{\left[\frac{-22}{1050\times0.03\times580}\times\tau\right]}$$

$$\ln\left[\frac{528-923}{313-923}\right] = \frac{22}{1050\times0.03\times580}\times\tau$$

$$\tau = 360.8 \text{ sec}$$

17. A aluminium sphere mass of 5.5 kg and initially at a temperature of 290^{o}Cis suddenly immersed in a fluid at 15 ^{o}C with heat transfer co efficient 58 W/m^2 K. Estimate the time required to cool the aluminium to 95^o C for aluminium take ρ = 2700 kg/m^3 , c = 900 J /kg K, k = 205 W/mK.

Given:

M = 5.5 kg

T_o = 290 + 273 = 563 K

T_∞ = 15 + 273 = 288 K

T = 95 + 273 =368 K

h = 58 W/m^{2}K

k = 205 W/mK

ρ = 2700 kg/m^3

c = 900 j/kg K

To find:

Time required to cool at 95^o C (τ)

Solution

$$\text{Density} = \rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{m}}{\text{v}}$$

$$V = \frac{m}{p} = \frac{5.5}{2700}$$

$$V = 2.037 \text{ X } 10^{-33}$$

For sphere,

$$\text{Characteristic length} \quad L_c = \frac{R}{3}$$

$$\text{Volume of sphere} \quad V = \frac{4}{3}\pi R^3$$

$$R = \sqrt[3]{\frac{3V}{4\pi}}$$

$$= \sqrt[3]{\frac{3\times 2.03\times 10^{-3}}{4\pi}}$$

$$R = 0.0786 \text{ m}$$

$$L_c = \frac{0.0786}{3} = 0.0262\ m$$

$$\text{Biot number } B_i = \frac{hL_c}{k}$$

$$= \frac{58 \times 0.0262}{205}$$

$$B_i = 7.41 \text{ X } 10^{-3} < 0.1$$

B_i < 0.1 this is lumped heat analysis type problem.

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{\left[\frac{-h}{c\,L_c \rho} \times \tau\right]}$$

$$\frac{368 - 288}{536 - 288} = e^{\left[\frac{58}{900 \times 0.0262 \times 2700} \times \tau\right]}$$

$$\tau = 1355.4 \text{ sec}$$

Unit II

May 2012

1. Air at 25 ^oC flows past a flat plate at 2.5 m/s. the plate measures 600 mm X 300 mm and is maintained at a uniform temperature at 95 ^oC. Calculate the heat loss from the plate, if the air flows parallel to the 600 mm side. How would this heat loss be affected if the flow of air is made parallel to the 300 mm side.

Given:

Forced convection (air)

Flat plate

$T_\infty = 25^o$ C

U = 25 m/s

$T_w = 95\ ^oC$

L = 600 mm = 600 X 10^{-3} m

W = 300 mm = 300 X 10^{-3} m

Find

(i) Q if air flows parallel to 600 mm side

(ii) Q if air flows parallel to 300 mm side and % of heat loss.

Solution:

$$T_f = \frac{T_w - T_\infty}{2} = \frac{95 - 25}{2} = \frac{120}{2} = 60^o\ C$$

Take properties of air at $T_f = 60^o$ C from H.M.T data book (page no 34)

Pr = 0.696

$\gamma = 1897 \times 10^{-6}\ m^2/s$

k = 0.02896

$$Re = \frac{UL}{\gamma} = \frac{2.5 \times 0.6}{18.97 \times 10^{-6}}$$

$$Re = 7.91 \times 10^4 < 5 \times 10^5$$

This flow is laminar.

From H.M.T data book

$$Nu_x = 0.332 {Re_x}^{0.5}\, pr^{0.333}$$

(or) $$Nu_L = 0.332 {Re_L}^{0.5}\, pr^{0.333}$$

$= 0.332 \text{ X } (7.91 \text{ X } 10^{4})^{0.5} (0.696)^{0.333}$

$Nu_L = 82.76$

$$\overline{N_u} = 2Nu_L = 2 \times 82.76$$

$$\overline{N_u} = 165.52$$

$$\overline{N_u} = \frac{\bar{h}L}{k}$$

$$h\,(or)\bar{h} = \frac{\overline{N_u}k}{L} = \frac{165.52 \times 0.02896}{0.6}$$

$$h\,(or)\bar{h} = 7.989\ W/m^2K$$

$$Q = \bar{h}A(\Delta T)(or)h(w.L)(T_w - T_\infty)$$

$$Q_1 = 7.989\,(0.6 \times 0.3)(95 - 25)$$

$Q_1 = 100.66$ W

(iii) If L = 0.3 m and W = 0.6 m (parallel to 300 mm side)

$$R_e = \frac{UL}{\gamma} = \frac{2.5 \times 0.3}{18.97 \times 10^{-6}} = 3.95 \times 10^4$$

$$R_e = 3.95 \times 10^4 < 5 \times 10^5$$

$$the\ flow\ is\ laminar$$

From H.M.T Data book

$$Nu_x = 0.332x^{0.5}Pr^{0.333}$$

$$(or)Nu_L = 0.332{Re_L}^{0.5}Pr^{0.333}$$

$$Nu_L = 0.332(3.95 \times 10^4)^{0.5}(0.696)^{0.333}$$

$Nu_L = 58.48$

$$\overline{Nu} = 2Nu_L = 2 \times 58.48 = 116.96$$

$$\overline{N_u} = \frac{\bar{h}L}{k}$$

$$\bar{h} = \frac{\overline{N_u}k}{L} = \frac{116.96 \times 0.02896}{0.3}$$

$$h\,(or)\bar{h} = 11.29\ W/m^2K$$

$$Q_2 = hA(\Delta T)(or)h(w.L)(T_w - T_\infty)$$

$$Q_2 = 11.29\,(0.6 \times 0.3)(95 - 25)$$

$Q_2 = 142.25$W

$\%\ \text{heat loss} = \frac{Q_2 - Q_1}{Q_1} \times 100$

$= \frac{142.25 - 100.66}{100.66} \times 100$

$$\%\ \text{heat loss} = 41.32\%$$

2. When 0.6 kg of water per minute is passed through a tube of 2 cm diameter, it is found to be heated from 20^{o}C to 60^{o}C. the heating is achieved by condensing steam on the surface of the tube and subsequently the surface temperature of the tube is maintained at 90^o C. Determine the length of the tube required for fully developed flow.

Given:

Mass, m = 0.6kg/min = 0.6/60 kg/s

= 0.01 kg/s

Diameter, D = 2 cm = 0.02m

Inlet temperature, T_{mi} = 20^o C

Outlet temperature, T_{mo} = 60^{o}C

Tube surface temperature , T_w= 90^{o}C

To find

length of the tube,(L).

Solution:

Bulk mean temperature = $T_m = \frac{T_{mi}+T_{mo}}{2} = \frac{20+60}{2} = 40^o C$

Properties of water at 40^{o}C:

(From H.M.T Data book, page no 22, sixth edition)

P = 995 kg/m^3

V = 0.657x10^{-6} m^2/s

Pr = 4.340

K = 0.628W/mK

C_p = 4178J/kgK

Mass flow rate, $\dot{m} = \rho AU$

$$U = \frac{\dot{m}}{\rho A}$$

$$U = \frac{0.01}{995 \times \frac{\pi}{4}(0.02)^2}$$

$$\text{velocity}, U = 0.031\text{m/s}$$

Let us first determine the type of flow

$$Re = \frac{UD}{v} = \frac{0.031 \times 0.02}{0.657\text{x}10^{-6}}$$

$$Re = 943.6$$

Since Re < 2300, the flow is laminar.

For laminar flow,

Nusselt Number, Nu = 3.66

We know that

$$Nu = \frac{hD}{k}$$

$$3.66 = \frac{h \times 0.02}{0.628}$$

$$h = 114.9\ W/m^2K$$

Heat transfer, $Q = mc_p \Delta T$

$$Q = mc_p(T_{mo} - T_{mi})$$

$$= 0.01 \times 4178 \times (60 - 20)$$

Q = 1671.2 W

We know that $Q = hA\Delta T$

$$Q = h \times \pi \times D \times L \times (T_w - T_m)$$

$$1671.2 = 114.9 \times \pi \times 0.02 \times L \times (90 - 40)$$

Length of tube , L = 4.62m

November 2012

3. Water is to be boiled at atmospheric pressure in a polished copper pan by means of an electric heater. The diameter of the pan is 0.38 m and is kept at 115^o C. calculate the following

1. Surface heat flux
2. Power required to boil the water
3. Rate of evaporation
4. Critical heat flux

Given:

Diameter, d = 0.38 m

Surface temperature, T_w = 115^{o}C

To find

1.Q/A

2. P

3. $\dot{m}$

4. $(Q/A)_{max}$

Solution:

We know that, Saturation temperature of water is 100^o C

i.e. T_{sat} = 100^{o}C

Properties of water at 100^oC:

(From H.M.T Data book, page no 22, sixth edition)

Density, ρ_l = 961 kg/m^3

Kinematic viscosity, v = 0.293×10^{-6} m^2/s

Prandtl Number, Pr = 1.740

Specific heat, C_{pl} = 4216 J/kgK

Dynamic viscosity, $\mu_l = \rho_l \times v = 961 \times 0.293 \times 10^{-6}$

$= 281.57 \text{ X } 10^{-6}$ Ns/m^2

From Steam table [R.S khurmi steam table]

At 100^o C

Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg.

$h_{fg} = 2256.9 \times 10^3$ J/kg

Specific volume of vapour, v_g = 1.673 m^3/kg

Density of vapour, $\rho_v = \frac{1}{v_g}$

$$\rho_v = \frac{1}{1.673}$$

$$\rho_v = 0.597\ kg/m^3$$

$$\Delta T = excess\ temperature = T_w - T_{sat} = 115^o - 100^o = 15^oC$$

$$\Delta T = 15^oC < 50^oC. So\ this\ is\ Nucleate\ pool\ boiling\ process.$$

Power required to boil the water,

For $Nucleate\ pool\ boiling$

Heat flux, $\frac{Q}{A} = \mu_l \times h_{fg} \left[\frac{g\times(\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{Cpl \times \Delta T}{C_{sf} \times h_{fg} {P_r}^n}\right]^3$ ….(1)

(From H.M.T Data book)

Where $\sigma = surface\ tension\ for\ liquid\ vapour\ interface$

At 100^oC

$\sigma = 0.0588\ N/m$ (From H.M.T Data book)

For water – copper → C_{sf} = surface fluid constant = 0.013

N = 1 for water (From H.M.T Data book)

Substitute

$\mu_l, h_{fg}, \rho_l, \rho_v, \sigma, Cpl, \Delta T, C_{sf}, n, h_{fg}, p_r$ values in eqn (1)

$$\frac{Q}{A} = 281.57 \times 10^{-6} \times 2256.9 \times 10^{3\times} \left[\frac{9.81 \times (961 - 0.597)}{0.0588}\right]^{0.5}$$

$$\times \left[\frac{4216 \times 15}{0.013 \times 2256.9 \times 10^3 \times (1.74)^1}\right]^3$$

$Surface\ Heat\ flux\ , \frac{Q}{A} = 4.83 \times 10^5 W/m^2$

$$Heat\ transfer,, Q = 4.83 \times 10^5 \times A$$

$$= 4.83 \times 10^5 \times \frac{\pi}{4} d^2$$

$$= 4.83 \times 10^5 \times \frac{\pi}{4} (0.38)^2$$

$$Q = 54.7\ x10^3\ W$$

$$Q = 54.7\ x10^3\ =P$$

$$Power = 54.7\ x10^3\ W$$

2. Rate of evaporation, ($\dot{m}$)

We know that,

$$\text{Heat transferred, } Q = \dot{m} \times h_{fg}$$

$$\dot{m} = \frac{Q}{h_{fg}} = \frac{54.7 \times 10^3}{2256.9 \times 10^3}$$

$$\dot{m} = 0.024\ kg/s$$

3. Critical heat flux, (Q/A)

$For\ Nucleate\ pool\ boiling, critical\ heat\ flux,$

$$\frac{Q}{A} = 0.18 h_{fg} \times \rho_v \left[\frac{\sigma \times g \times (\rho_l - \rho_v)}{{\rho_v}^2}\right]^{0.25}$$

(From H.M.T Data book)

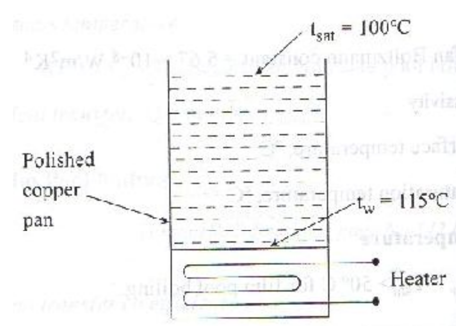
$$= 0.18 \times 2256.9 \times 10^3 \times 0.597 \times \left[\frac{0.0588 \times 9.81 \times (961 - 0.597)}{(0.597)^2}\right]^{0.25}$$

$$Critical\ heat\ flux\ , q = \frac{Q}{A} = 1.52 \times 10^6 W/m^2$$

May 2013

4. A thin 80 cm long and 8 cm wide horizontal plate is maintained at a temperature of 130°C in large tank full of water at 70°C. Estimate the rate of heat input into the plate necessary to maintain the temperature of 130°C.

Given:



Horizontal plate length, L = 80 cm = 0.08m

Wide, W = 8 cm = 0.08 m,

Plate temperature, T_w = 130°C

Fluid temperature, T_∞ = 70°C

To find:

Rate of heat input into the plate,Q.

Solution:

Flim temperature, $T_f = \frac{T_w - T_\infty}{2} = \frac{130+70}{2} = 100^o\ C$

Properties of water at 100°C:

(From H.M.T Data book, page no 22, sixth edition)

ρ= 961 kg/m^3

v = 0.293x10^{-6} m^2/s

Pr = 1.740

k = 0.6804W/mK

$\beta_{water} = 0.76 \times 10^{-3} K^{-1}$

(From H.M.T Data book, page no 30, sixth edition)

We know that,

$$Grashof\ number, Gr = \frac{g \times \beta \times {L_c}^3 \times \Delta T}{V^2}$$

For horizontal plate:

$$\mathrm{L_c} = \text{Characteristic length} = \frac{W}{2}$$

$$\mathrm{L_c} = \frac{0.08}{2}$$

$$\mathrm{L_c} = 0.04\ m$$

$$Grashof\ number, Gr = \frac{9.81 \times 0.76 \times 10^{-3} \times (0.04)^3 \times (130 - 70)}{(0.293 \times 10^{-6})^2}$$

$$Gr = 0.333 \times 10^9$$

$$GrPr = 0.333 \times 10^9 \times 1.740$$

$$GrPr = 0.580 \times 10^9$$

GrPr value is in between 8x10^6 and 10^{11}

i.e., 8x10^6 < GrPr<10^{11} So, for horizontal plate, upper surface heated,

Nusselt number, Nu = 0.15(GrPr)$^{0.333}$

(From H.M.T Data book, page no 136, sixth edition)

Nu = 0.15(0580 x 10^9)$^{0.333}$

Nu = 124.25

$$\text{Nusselt number, Nu} = \frac{\mathrm{h_u L_c}}{\mathrm{k}}$$

$$124.25 = \frac{\mathrm{h_u} \times 0.04}{0.6804}$$

$$\mathrm{h_u} = 2113.49\ \mathrm{W/m^2K}$$

Heat transfer coefficient for upper surface heated h_u = 2113.49 $\mathrm{W/m^2K}$

For horizontal plate, Lower surface heated:

Nusselt number, Nu_l = 0.27(GrPr)$^{0.25}$

(From H.M.T Data book, page no 137, sixth edition)

$$= 0.27[0.580 \times 10^9]^{0.25}$$

$$Nu_l = 42.06$$

We know that,

$$\text{Nusselt number}, Nu_l = \frac{h_l L_c}{k}$$

$$42.06 = \frac{h_l \times 0.04}{0.6804}$$

$$h_l = 715.44 \text{ W/m}^2\text{K}$$

Heat transfer coefficient for lower surface heated $h_l = 715.44 \text{ W/m}^2\text{K}$

Total heat transfer, $Q = (h_u + h_l)A\,\Delta T$

$$= (h_u + h_l) \times W \times L \times [T_w - T_\infty]$$

$$= (2113.49 + 715.44) \times (0.08 \times 0.8) \times [130 - 70]$$

$$Q = 10.86 \times 10^3 W$$

5. A vertical pipe 80 mm diameter and 2 m height is maintained at a constant temperature of 120 ^{o} C. the pipe is surrounded by still atmospheric air at 30^o . Find heat loss by natural convection.

Given:

Vertical pipe diameter D = 80 mm = 0.080m

Height (or) length L = 2 m

Surface temperature $T_S = 120\,^o C$

Air temperature $T_\infty = 30\,^o C$

To find

heat loss (Q)

Solution:

We know that

Flim temperature , $T_f = \frac{T_w + T_\infty}{2} = \frac{120+30}{2} = 75^o\, C$

Properties of water at 75 ^{o}C:

ρ= 1.0145 kg/m^3

$\nu = 20.55 \times 10^{-6}$ m^2/s

Pr = 0.693

$k = 30.06 \times 10^{-3}$ W/mK

We know

$$\beta = \frac{1}{T_f in\ K}$$

$$\beta = \frac{1}{75+273} = 2.87 \times 10^{-3} K^{-1}$$

We know

$$Grashof\ number, Gr = \frac{g \times \beta \times L^3 \times \Delta T}{V^2}$$

$$= \frac{9.81 \times 2.87 \times 10^{-3} \times (0.08)^3 \times (120-30)}{(20.55 \times 10^{-6})^2}$$

$$Gr = 4.80 \times 10^{10}$$

$$GrPr = 4.80 \times 10^{10} \times 0.693$$

$$GrPr = 3.32 \times 10^{10}$$

Since GrPr>10^9, flow is turbulent.

For turbulent flow, from HMT data book

$$Nu = 0.10(GrPr)^{0.333}$$

$$Nu = 0.10(3.32 \times 10^{10})^{0.333}$$

Nu = 318.8

We know that,

$$Nusselt\ number, Nu = \frac{hL}{k}$$

$$318.8 = \frac{h \times 2}{30.06 \times 10^{-3}}$$

$$Heat\ transfer\ cofficient, h = 4.79\ W/m^2 K$$

Heat loss, $Q = h \times A \times \Delta T$

$$= h \times \pi \times D \times L \times (T_s - T_\infty)$$

$$= 4.79 \times \pi \times 0.080 \times 2 \times (120-30)$$

Q = 216.7 W

Heat loss Q = 216.7.

November 2012

6. Derive an equation for free convection by use of dimensional analysis.

$$Nu = C(Pr^n . Gr^m)$$

Assume, h = f {ρ, μ, Cp, k, Σ,(β, ΔT)}

The heat transfer co efficient in case of natural or free convection, depends upon the variables, V, ρ, k,μ, Cp and L, or D. Since the fluid circulation in free convection is owing to difference in density between the various fluids layers due to temperature gradient and not by external agency.

Thus heat transfer coefficient 'h' may be expressed as follows:

$h = f(\rho, L, \mu, c_p, k, \beta\, g\, \Delta T)$ ……….(i)

$f_1\,(\rho, L, \mu, k, h, c_p, \beta\, g\, \Delta T)$ ……….(ii)

[This parameter (β g ΔT) represents the buoyant force and has the dimensions of LT^{-2}.]

Total number of variables, n = 7

Fundamental dimensions in the problem are M,L,T, θ and hense m = 4

Number of dimensionless π- terms = (n –m) = 7-4= 3

The equation (ii) may be written as

$$f_1\,(\pi_1, \pi_2, \pi_3) = 3$$

We close ρ, L, μ and k as the core group (repeating variables) with unknown exponents. The groups to be formed are now represented as the following π groups.

$$\pi_1 = \rho^{a_1}.L^{b_1}.\mu^{c_1}.k^{d_1}.h$$

$$\pi_2 = \rho^{a_2}.L^{b_2}.\mu^{c_2}.k^{d_2}.c_p$$

$$\pi_3 = \rho^{a_3}.L^{b_3}.\mu^{c_3}.k^{d_3}.\beta\; g\; \Delta t$$

π_1- term:

$$M^O L^0 T^0 \theta^O = (ML^{-3})^{a_1}.(L)^{b_1}.(ML^{-1}T^{-1})^{c_1}.(MLT^{-3}\theta^{-1})^{d_1}.(ML^{-3}\theta^{-1})$$

Equating the exponents of M,L,T and θ respectively, we get

For M: $0 = a_1 + c_1 + d_1 + 1$

For L: $0 = -3a_1 + b_1 - c_1 + d_1$

For T: $0 = -c_1 + 3d_1 - 3$

For T: $\theta = -d_1 - 1$

Solving the above equations, we get

$a_1 = 0, b_1 = 1, c_1 = 0, d_1 = -1$

$$\pi_1 = Lk^{-1}h\ (or)\ \pi_{1=} \frac{hL}{k}$$

π_2- Term:

$$M^O L^0 T^0 \theta^O = (ML^{-3})^{a_2}.(L)^{b_2}.(ML^{-1}T^{-1})^{c_2}.(MLT^{-3}\theta^{-1})^{d_2}.(L^2T^{-2}\theta^{-1})$$

Equating the exponents of M, L, T and θ respectively, we get

For M: $0 = a_2 + c_2 + d_2$

For L: $0 = -3a_2 + b_2 - c_2 + d_2 + 2$

For T: $0 = -c_2 - 3d_2 - 2$

For T: $\theta = -d_2 - 1$

Solving the above equations, we get

$a_2 = 0, b_2 = 0, c_2 = 1, d_2 = -1$

$$\pi_2 = \mu.k^{-1}.c_p\ (or)\pi_2 = \frac{\mu c_p}{k}$$

π_3- Term:

$$M^O L^0 T^0 \theta^O = (ML^{-3})^{a_3}.(L)^{b_3}.(ML^{-1}T^{-1})^{c_3}.(MLT^{-3}\theta^{-1})^{d_3}.(LT^{-2})$$

Equating the exponents of M, L, T and θ respectively, we get

For M: $0 = a_3 + c_3 + d_{32}$

For L: $0 = -3a_3 + b_3 - c_3 + d_3 + 1$

For T: $0 = -c_3 - 3d_3 - 2$

For T: $\theta = -d_3$

Solving the above equations, we get

$a_3 = 2, b_3 = 3, c_3 = -2, d_3 = 0$

$$\pi_3 = \rho^2.L^3\mu^{-2}.(\beta g\Delta t)$$

$$or \qquad \pi_3 = \frac{(\beta g\Delta t)\rho^2.L^3}{\mu^2} = \frac{(\beta g\Delta t)L^3}{v^2}$$

$$or \qquad Nu = \emptyset(Pr)(Gr)$$

$$or \qquad Nu = C(Pr)^n(Gr)^m(where\ Gr = Grashoff\ number)$$

Here C, n and m are constants and may be evaluated experimentally.

UNIT - III

1. Two large plates are maintained at a temperature of 900 K and 500 K respectively. Each plate has area of 6^2. Compare the net heat exchange between the plates for the following cases.

(i) Both plates are black

(ii) Plates have an emissivity of 0.5

Given:

$T_1 = 900$ K

$T_2 = 500$ K

$A = 6\ m^2$

To find:

(i) $(Q_{12})_{net}$ Both plates are black $\epsilon = 1$

(ii) $(Q_{12})_{net}$ Plates have an emissivity of $\epsilon = 0.5$

Solution

Case (i) $\epsilon_1 = \epsilon_2 = 1$

$$(Q_{12})_{net} = \frac{A\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(Q_{12})_{net} = \frac{A \times 5.67\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(Q_{12})_{net} = \frac{6 \times 5.67\left[\left(\frac{900}{100}\right)^4 - \left(\frac{500}{100}\right)^4\right]}{\frac{1}{1} + \frac{1}{1} - 1}$$

$$(Q_{12})_{net} = 201.9 \times 10^3 W$$

Case (ii) $\epsilon_1 = \epsilon_2 = 0.5$

$$(Q_{12})_{net} = \frac{A\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(Q_{12})_{net} = \frac{6 \times 5.67\left[\left(\frac{900}{100}\right)^4 - \left(\frac{500}{100}\right)^4\right]}{\frac{1}{0.5} + \frac{1}{0.5} - 1}$$

$$(Q_{12})_{net} = 67300\ W$$

2. **The sun emits maximum radiation at λ = 0.52 μ. Assuming the sun to be a black body, calculate the surface temperature of the sun. Also calculate the monochromatic emissive power of the sun's surface.**

Given:

$\lambda_{max} = 0.52\ \mu = 0.52 \times 10^{-6}$ m

To find:

(i) Surface temperature, T.

(ii) Monochromatic emissive power, $E_{b\lambda}$

(iii) Total emissive power, E

(iv) Maximum emissive power, E_{max}

Solution:

1. **From Wien's law,**

$\lambda_{max}\ T = 2.9 \times 10^{-3}$ mK

[From HMT Data book, page no 82, sixth editions]

$$T = \frac{2.9 \text{ x } 10 - 3}{0.52 \text{ x } 10 - 6}$$

$$T = 5576\ K$$

2. **Monochromatic emissive power, ($E_{b\lambda}$)**

From Planck's law,

$$E_{b\lambda} = \frac{c_1 \lambda^{-5}}{\left[e^{\left(\frac{c_2}{\lambda T}\right)} - 1\right]}$$

[From HMT Data book, page no 82, sixth editions]

Where

$$c_1 = 0.374 \times 10^{-15}\ Wm^2$$

$$c_2 = 14.4 \times 10^{-3}\ mK$$

$$\lambda = 0.52 \text{ x } 10^{-6} \text{ m}$$

T = 5576 K

$$E_{b\lambda} = \frac{0.374 \times 10^{-15}[0.52 \text{ x } 10^{-6}]^{-5}}{\left[e^{\left(\frac{14.4 \times 10^{-3}}{0.52 \text{ x } 10^{-6} \times 5576}\right)} - 1\right]}$$

$$E_{b\lambda} = 6.9 \times 10^{13}\ \text{W/m}^2$$

3. Total emissive power

$E = \sigma T^4 = 5.67 \times 10^{-6} \times (5576)^4$ W/m^2

4. Maximum emissive power

$E_{max} = 1.285 \times 10^{-5}\ T^5 = 1.285 \times 10^{-5} (5576)^5$ W/m^2

3. A 70 mm thick metal plate with a circular hole of 35 mm diameter along the thickness is maintained at a uniform temperature 250 ^{o} C. Find the loss of energy to the surroundings at 27 ^{o}, assuming the two ends of the hole to be as parallel discs and the metallic surfaces and surroundings have black body characteristics.

Given:

$$r_2 = (r_{3)} = \frac{35}{2} = 17.5\,mm = 0.0175\,m$$

L = 70 mm =0.07 m

T_1 = 250 +273 = 523 K

T_{surr} = 27 +273 = 300 K

Let suffix 1 designate the cavity and the suffices 2 and 3 denote the two ends of 35 mm dia. Hole which are behaving as discs. Thus,

$$\frac{L}{r_2} = \frac{0.07}{0.0175} = 4$$

$$\frac{r_3}{L} = \frac{0.0175}{0.07} = 0.25$$

The configuration factor, $F_{2\text{-}3}$ is 0.065

Now, $F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1$ …….By summation rule

But, $F_{2\text{-}2} = 0$

$F_{2\text{-}1} = 1 - F_{2\text{-}3} = 1 – 0.065 = 0.935$

Also,

$A_1 F_{1\text{-}2} = A_2 F_{2-1}$ .....By reciprocating theorem

$$F_{1-2} = \frac{A_2 F_{2-1}}{A_1} = \frac{\pi\ \times (0.0175)^2 \times 0.935}{\pi\ \times 0.035 \times 0.07} = 0.1168$$

$F_{1-3} = F_{1-2} = 0.1168$ ………. By symmetry

$\boldsymbol{The\ total\ loss\ of\ energy = loss\ of\ heat\ by\ both\ ends}$

$$= \mathrm{A_1\ F_{1-2}\, \sigma \left(T_1{}^4 - T_{surr}{}^4\right) + A_1\ F_{1-3}\, \sigma \left(T_1{}^4 - T_{surr}{}^4\right)}$$

therefore ($\mathrm{F_{1-2} = F_{1-3}}$)

$$= 2\ \mathrm{A_1\ F_{1-2}\, \sigma \left(T_1{}^4 - T_{surr}{}^4\right)}$$

$$= 2\ (\pi\ \times 0.035 \times 0.07) \times 0.1168 \times 5.6 \left[\left(\frac{523}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right] = 6.8\ \mathrm{W}$$

November 2011

4. The filament of a 75 W light bulb may be considered as a black body radiating into a black enclosure at 70^0 C. the filament diameter is 0.10 mm and length is 5 cm. considering the radiation, determine the filament temperature .

Given:

$Q = 75W = 75\ J/s$

$T_2 = 70 + 273 = 343\ K$

$d = 0.1\ mm$

$l = 5\ cm$

Area = $\pi\ dl$

Solution:

$\epsilon = 1$ for black body

$$Q = \sigma\epsilon A\left({T_1}^4 - {T_2}^4\right)$$

$$75 = 5.67 \times 10^{-8} \times 1 \times \pi \times 0.1 \times 10^{-3} \times 5 \times 10^{-2}\left({T_1}^4 - (343)^4\right)$$

$${T_1}^4 = \frac{75}{8.906 \times 10^{-13}} + (343)^4$$

$$T_1 = 3029\ K$$

$$T_1 = 3029 - 273 = 2756^0C$$

November 2011 (old regulation)

5. **Two parallel plates of size 1.0 m by 1.0 m spaced 0.5 m apart are located in a very large room, the walls of which are maintained at a temperature of 27^0 C. one p[late is maintained at a temperature of 900^0 C and other at 400^0C. their emissivities are 0.2 and 0.5 respectively. If the plates exchange heat between themselves and the surroundings, find the net heat transfer to each plate and to the room. Consider only the plate surface facing each other.**

Given:

Three surfaces (2 plates and wall)

$$T_1 = 900^0C = 1173\ K$$

$$T_2 = 400^0C = 673\ K$$

$$T_3 = 27^0C = 300\ K$$

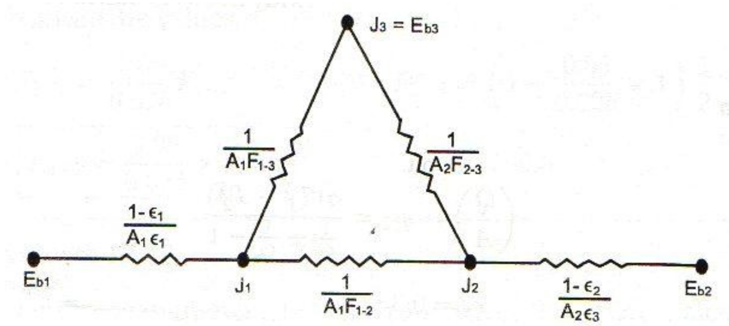
$$A_1 = A_2 = 1.0\ m^2$$

$$\epsilon_1 = 0.2$$

$$\epsilon_2 = 0.2$$

Room size is much larger than the plate size

$$Surface\ resistance\ \frac{1-\epsilon_3}{\epsilon_3\ A_3} = 0\ and\ then\ E_{b3} = J_3$$



1. To find the shape factor F_{1-2}.

Ratio of smaller side to distance between plane.

$$= \frac{1}{0.5} = 2$$

Corresponding to 2 and curve 2 in HMT Data book

$$F_{1-2} = 0.4$$

By summation rule

$F_{1-2} + F_{1-3} = 1$

$F_{1-3} = 1 - F_{1-2}$

$F_{1-3} = 1 - 0.4 = 0.6$

$F_{1-3} = 0.6$

$F_{2-1} + F_{2-3} = 1$

$F_{2-3} = 1 - F_{2-1}$

$F_{2-3} = 1 - 0.4$

$F_{2-3} = 0.6$

The resistances are

$$R_1 = \frac{1-\epsilon_1}{\epsilon_1\ A_1} = \frac{1-0.2}{0.2 \times 1} = 4.0$$

$$R_2 = \frac{1-\epsilon_2}{\epsilon_2\ A_2} = \frac{1-0.5}{0.5 \times 1} = 1.0$$

$$R_{1-2} = \frac{1}{A_1 F_{1-2}} = \frac{1}{1 \times 0.4} = 1.0$$

$$R_{1-3} = \frac{1}{A_1 F_{1-3}} = \frac{1}{1 \times 0.6} = 1.67$$

$$R_{2-3} = \frac{1}{A_2 F_{2-3}} = \frac{1}{1 \times 0.6} = 1.67$$

To find radiosities $J_1 J_2$ and J_3, find total emissive power (E_b)

$$E_{b1} = \sigma T_1^{\ 4} = 5.67\left(\frac{1173}{100}\right)^4 = 107.4\ kW/m^2$$

$$E_{b2} = \sigma T_2^{\ 4} = 5.67\left(\frac{673}{100}\right)^4 = 11.7\ kW/m^2$$

$$E_{b3} = \sigma T_3^{\ 4} = 5.67\left(\frac{300}{100}\right)^4 = 0.46\ kW/m^2$$

Node J₁ :

$$\frac{E_{b1}-J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}} + \frac{J_2-J_1}{\frac{1}{A_1F_{1-2}}} + \frac{E_{b3}-J_1}{\frac{1-\epsilon_1}{A_1F_{1-3}}} = \frac{107.4-J_1}{4.0} + \frac{J_2-J_1}{2.5} + \frac{0.46-J_1}{1.67}$$

J_1 in terms of J_2

Node J₂

$$\frac{J_1-J_2}{R_{1-2}} + \frac{E_{b3}-J_2}{R_{2-3}} + \frac{E_{b2}-J_2}{R_2}$$

Here J_1 in terms of J_2

$J_2 = 11.6 kW/m^2$

And $J_1 = 25.0 kW/m^2$

The total heat loss by plate (1) is

$$Q_1 = \frac{E_{b1}-J_1}{\frac{1-\epsilon_1}{\epsilon_1\ A_1}} = \frac{107.4-25}{4.00} = 20.6\ kW$$

The total heat loss by plate (2) is

$$Q_1 = \frac{E_{b2}-J_2}{\frac{1-\epsilon_2}{\epsilon_2\ A_2}} = \frac{11.7-11.6}{1.00} = 0.1\ kW$$

The total heat received by the room is

$$Q_3 = Q_1 + Q_2$$

$$Q_3 = 20.6 + 0.1$$

$$Q_3 = 20.7\ kW$$

Net energy lost by the plates = Absorbed by the room.

6. **Two large parallel planes with emissivities of 0.3 and 0.5 are maintained at temperatures of 527^0 C and 127^0C respectively. A radiation shield having emissivities of 0.05 on both sides is placed between them. Calculate**
   (i) Heat transfer rate between them without shield.
   (ii) Heat transfer rate between them with shield.

Given:

$\epsilon_1 = 0.3$

$\epsilon_2 = 0.5$

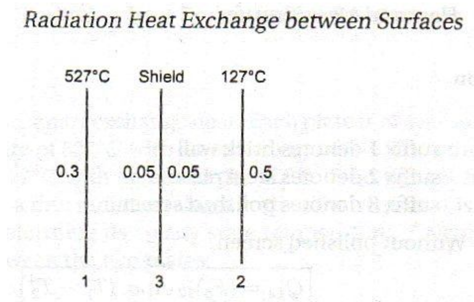
$\epsilon = 0.05$

$T_1 = 527 + 273 = 800$ K

T_2 = 127+ 273 = 400 K

Find:

$Q_{w/o\ shield}$ and $Q_{with\ shield}$



Solution:

$$(Q_{12})_{net\ without\ shield} = \frac{\sigma\left({T_1}^4 - {T_2}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{5.67\left(\left(\frac{800}{100}\right)^4 - \left(\frac{400}{100}\right)^4\right)}{\frac{1}{0.3} + \frac{1}{0.5} - 1}$$

$$(Q_{12})_{net\ without\ shield} = 5024.5\ W/m^2$$

$$(Q_{12})_{with\ shield} = \frac{\sigma\left({T_1}^4 - {T_2}^4\right)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right) + \left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right)}$$

$$= \frac{5.67(8^4 - 4^4)}{\left(\frac{1}{0.3} + \frac{1}{0.05} - 1\right) + \left(\frac{1}{0.05} + \frac{1}{0.5} - 1\right)}$$

$$(Q_{12})_{with\ shield} = 859.45\ W/m^2$$

November 2012

7. Emissivities of two large parallel plates maintained at 800^o C and 300^0 C are 0.3 and 0.5 respectively. Find the net radiant heat exchange per square meter of the plates. If a polished aluminium shield (Є = 0.05) is placed between them. Find the percentage of reduction in heat transfer.

Given:

T_1 = 800^o C +273 = 1073 K

T_2 = 300^o C +273 = 573 K

ε_1 = 0.3

ε_2 = 0.3

Radiation shield emissivity ε_3= 0.05

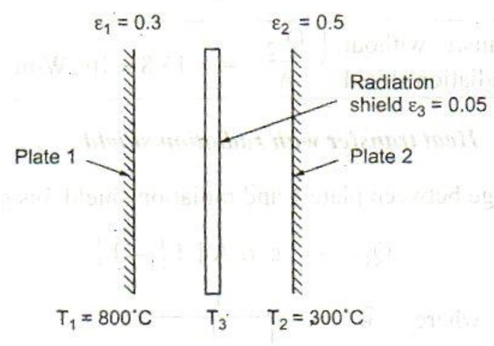


Fig. 4.27.

To find:

(i) Net radiant heat exchange per square meter $\left[\frac{Q_{12}}{A}\right]$

(ii) Percentage of reduction in heat transfer due to radiation shield.

Solution:

Case I: **Heat transfer without radiation shield**:

Heat exchange between two large parallel plates without radiation shield is given by

$$Q_{12} = \vec{\varepsilon}\sigma A\left[{T_1}^4 - {T_2}^4\right]$$

Where

$$\vec{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{1}{\frac{1}{0.3} + \frac{1}{0.5} - 1}$$

$$\vec{\varepsilon} = 0.230$$

$$Q_{12} = 0.230 \times 5.67 \times 10^{-8} \times A \times [(1073)^4 - (573)^4]$$

Heat transfer without radiation shield $\left[\frac{Q_{12}}{A}\right]$ **= 15.8 X10^{3}W/m^2**

Case II: **Heat transfer with radiation shield**:

Heat exchange between plate I and radiation shield 3 is given by

$$Q_{13} = \vec{\varepsilon}\sigma A\left[{T_1}^4 - {T_3}^4\right]$$

Where

$$\vec{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$Q_{13} = \frac{\sigma A\left[{T_1}^4 - {T_3}^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \qquad \ldots\ldots\ldots\ldots..(1)$$

Heat exchange between radiation shield 3 and plate 2 is given by

$$Q_{32} = \vec{\varepsilon}\sigma\, A\left[T_3{}^4 - T_2{}^4\right]$$

Where

$$\vec{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$Q_{32} = \frac{\sigma\, A\left[T_3{}^4 - T_2{}^4\right]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \qquad \ldots\ldots\ldots\ldots..(2)$$

We know that,

$$Q_{13} = Q_{32}$$

$$\frac{\sigma\, A\left[T_1{}^4 - T_3{}^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma\, A\left[T_3{}^4 - T_2{}^4\right]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{(1073)^4 - T_3{}^4}{\frac{1}{0.3} + \frac{1}{0.05} - 1} = \frac{T_3{}^4 - (573)^4}{\frac{1}{0.05} + \frac{1}{0.5} - 1}$$

$$= \frac{(1073)^4 - T_3{}^4}{22.3} = \frac{T_3{}^4 - (573)^4}{21}$$

$$= 2.78 \times 10^{13} - 21\, T_3{}^4 = 22.3\, T_3{}^4 - 2.4 \times 10^{12}$$

$$= 3.02 \times 10^{13} = 43.3\, T_3{}^4$$

Shield temperature $T_3 = 913.8\, K$

Heat transfer with radiation shield Q_{13} =

$$Q_{13} = \frac{\sigma\, A\left[T_1{}^4 - T_3{}^4\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$Q_{13} = \frac{5.67 \times 10^{-8} \times A \times [(1073)^4 - (913.8)^4]}{\frac{1}{0.3} + \frac{1}{0.05} - 1}$$

$$\frac{Q_{13}}{A} = 1594.6\ W/m^2 \ \ldots\ldots\ldots\ldots\ldots(3)$$

$$\%\ of\ reduction\ in\ heat\ transfer = \frac{Q_{without\ shield} - Q\ with\ shield}{Q_{without\ shield}}$$

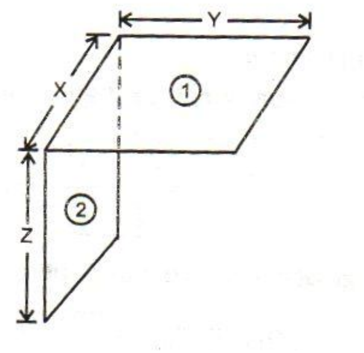
$$due\ to\ radiation\ shield$$

$$= \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{15.8 \times 10^3 - 1594.6}{15.8 \times 10^3}$$

$$= 0.899 = 89.9\ \%$$

8. Two rectangular surfaces are perpendicular to each other with a common edge of 2 m. the horizontal plane is 2 m long and vertical plane is 3 m long. Vertical plane is at 1200 K and has an emissivity of 0.4. the horizontal plane is 18^0 C and has a emissivity of 0.3. Determine the net heat exchange between the planes.



Solution:

$Q_{12} = ?$

$$Q_{12} = (Fg)_{1-2} A_1 \sigma \left({T_1}^4 - {T_2}^4 \right)$$

Here

$$(Fg)_{1-2} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{1-2}} + \left(\frac{1-\epsilon_2}{\epsilon_2}\right)\frac{A_1}{A_2}}$$

A_1 = Area of horizontal plane = XY = 2x2 = 4 m^2

A_2 = Area of vertical plane = ZX = 3x2 = 6 m^2

Both surfaces have common edge for which

$$\frac{Z}{X} = \frac{3}{2} = 1.5 \quad and \quad \frac{Y}{X} = \frac{2}{2} = 1$$

From HMT data book the shape factor $F_{1-2} = 0.22$

$$Q_{12=} \frac{4 \times 5.67 \left(\left(\frac{1200}{100}\right)^4 - \left(\frac{18+273}{100}\right)^4 \right)}{\frac{1-0.4}{0.4} + \frac{1}{0.22} + \left(\frac{1-0.3}{0.3}\right)\frac{4}{6}}$$

$$Q_{12=} \ 61657.7 \ W$$

9. Determine the view factor (F_{14}) for the figure shown below.

From Fig. We know that

$A_5 = A_1 + A_2$

$A_6 = A_3 + A_4$

Further,

$A_5 F_5 = A_1 F_{1-6} + A_2 F_{2-6}$

$[\because A_5 = A_1 + A_2; F_{5-6} = F_{1-6} + F_{2-6}]$

$= \quad A_1 F_{1-3} + A_1 F_{1-4} + A_2 F_{2-6}$

$[\because A_5 = A_1 + A_2;\ F_{5-6} = F_{1-6} + F_{2-6}]$

$A_5 F_{5-6} = A_5 F_{5-3} - A_2 F_{2-3} + A_1 F_{1-4} + A_2 F_{2-6}$

$[\because A_1 = A_5 + A_2;\ F_{1-3} = F_{5-3} - F_{2-3}]$

$\Rightarrow A_1 F_{1-4} = A_5 F_{5-6} - A_5 F_{5-3} + A_2 F_{2-3} - A_2 F_{2-6}$

$$\Rightarrow F_{1-4} = \frac{A_5}{A_1}[F_{5-6} - F_{5-3}] + \frac{A_2}{A_1}[F_{2-3} - F_{2-6}] \qquad ......(1)$$

[Refer HMT Data book, page No.94 (sixth Edition)

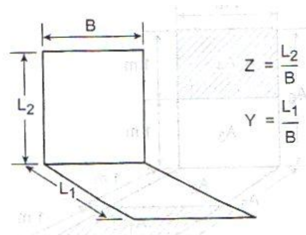


Fig. 4.52.

Shape factor for the area A_5 and A_6

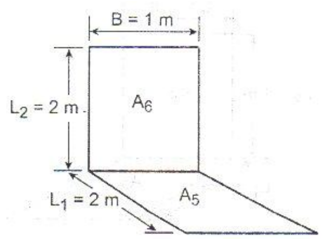


Fig. 4.53.

$$Z = \frac{L_2}{B} = \frac{2}{1} = 2$$

$$Y = \frac{L_1}{B} = \frac{2}{1} = 2$$

Z value is 2, Y value is 2. From that, we can find corresponding shape factor value is 0.14930. (From tables)

$F_{5-6} = 0.14930$

Shape factor for the area A_5 and A_3

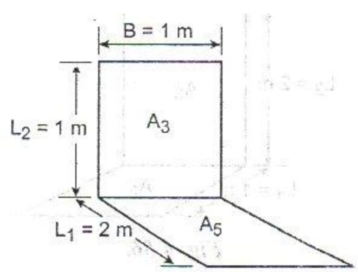


Fig. 4.54.

$$Z = \frac{L_2}{B} = \frac{1}{1} = 1$$

$$Y = \frac{L_1}{B} = \frac{2}{1} = 2$$

F_{5-3} = 0.11643

Shape factor for the area A_2 and A_3

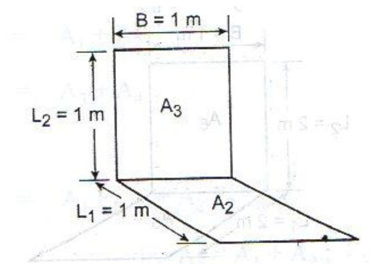


Fig. 4.55.

$$Z = \frac{L_2}{B} = \frac{1}{1} = 1$$

$$Y = \frac{L_1}{B} = \frac{1}{1} = 1$$

F_{2-3} = 0.20004

Shape factor for the area A_2 and A_6

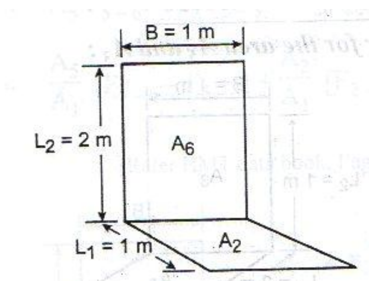


Fig. 4.56.

$$Z = \frac{L_2}{B} = \frac{2}{1} = 1$$

$$Y = \frac{L_1}{B} = \frac{1}{1} = 1$$

F_{2-6} = 0.23285

Substitute F_{5-6}, F_{5-3}, F_{2-3}, and F_{2-6} values in equation (1),

$$\Rightarrow F_{1-4} = \frac{A_5}{A_1}[0.14930 - 0.11643] + \frac{A_2}{A_1}[0.20004 - 0.23285]$$

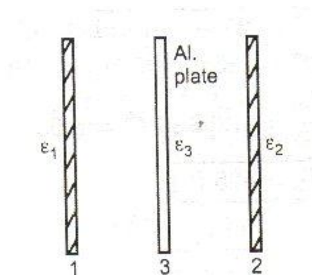
$$= \frac{A_5}{A_1}[0.03287] - \frac{A_2}{A_1}[0.03281]$$

$$F_{1-4} = 0.03293$$

Result :

View factor, $F_{1\text{-}4} = 0.03293$

10. Calculate the net radiant heat exchange per m^2 area for two large parallel plates at temperatures of 427^0 C and 27^{0}C. ϵ $_{(hot\ plate)}$ = 0.9 and ϵ $_{(cold\ plate)}$ = 0.6.If a polished aluminium shield is placed between them, find the % reduction in the heat transfer ϵ $_{(shield)}$ = 0.4



Net radiation heat transfer $(Q_{12})_{net}$ = ?

Given:

$T_1 = 427 + 273 = 700$ K

$T_2 = 27 + 273 = 300$ K

$\epsilon_1 = 0.9$

$\epsilon_2 = 0.6$

$\epsilon = 0.4$

Solution:

$$(Q_{12})_{net\ without\ shield} = \frac{\sigma(T_1^{\ 4} - T_2^{\ 4})}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{5.67\left(\left(\frac{700}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right)}{\frac{1}{0.9} + \frac{1}{0.6} - 1}$$

$$(Q_{12})_{net} = 7399.35\ W/m^2$$

Percentage reduction in the heat transfer flow

$$= \frac{Reduction\ in\ heat\ flow\ due\ to\ shield}{Net\ heat\ flow} \times 100$$

$$Reduction\ in\ heat\ flow\ due\ to\ shield = (Q_{12})_{net} - (Q_{13})_{net}$$

$$(Q_{13})_{net\ with\ shield} = \frac{A\sigma\left({T_1}^4 - {T_3}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1}$$

To find T_3 shield temperature $(Q_{13})_{net} = (Q_{32})_{net}$

$$\frac{A\sigma\left({T_1}^4 - {T_3}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{A\sigma\left({T_3}^4 - {T_2}^4\right)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1}$$

Let $\frac{T_3}{100} = x$

$$\frac{\left(\left(\frac{700}{100}\right)^4 - \left(\frac{T_3}{100}\right)^4\right)}{\frac{1}{0.9} + \frac{1}{0.4} - 1} = \frac{\left(\left(\frac{T_3}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right)}{\frac{1}{0.4} + \frac{1}{0.6} - 1}$$

$$\frac{2401 - x^4}{1.11 + 25 - 1} = \frac{x^4 - 81}{25 + 1.67 - 1}$$

$$x^4 = 1253.8$$

$$\frac{T_3}{100} = (1253.8)^{1/4} = 5.95 \qquad (or)$$

$$T_3 = 595\ K$$

$$(Q_{13})_{net} = \frac{\sigma\left({T_1}^4 - {T_3}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1}$$

$$= \frac{5.67\left(\left(\frac{700}{100}\right)^4 - \left(\frac{595}{100}\right)^4\right)}{\frac{1}{0.9} + \frac{1}{0.4} - 1}$$

$$(Q_{13})_{net} = 2492.14\ W/m^2$$

$$Reduction\ in\ heat\ flow\ due\ to\ shield = (Q_{12})_{net} - (Q_{13})_{net}$$

= 7399.35 -2492.14

= 4907.21 W/m^2

Percentage reduction = $\frac{4907.21}{7399.35}\ x\ 100 = 66.32\%$

11. There are two large parallel plane with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction when an aluminium shield of emissivity 0.04 is p[laced between them. Use the method of electrical analogy.

Solution:

Given:

$\epsilon_1 = 0.3$

$\epsilon_2 = 0.8$

$\epsilon = 0.04$

Percentage reduction in heat transfer

$$= \frac{Reduction\ in\ heat\ transfer\ due\ to\ shield}{Net\ heat\ transfer\ rate} \times 100$$

$$Reduction\ in\ heat\ flow\ due\ to\ shield = \frac{(Q_{12})_{net} - (Q_{13})_{net}}{(Q_{12})_{net}}$$

$$(Q_{12})_{net\ w/o\ shield} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\frac{1}{0.3} + \frac{1}{0.8} - 1} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{3.58}$$

$$(Q_{13})_{net\ with\ shield} = \frac{\sigma(T_1{}^4 - T_3{}^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{\sigma(T_1{}^4 - T_3{}^4)}{\frac{1}{0.3} + \frac{1}{0.04} - 1} = \frac{\sigma(T_1{}^4 - T_3{}^4)}{27.33}$$

Percentage reduction in heat transfer

$$= 1 - \frac{(Q_{13})}{(Q_{12})}$$

Here T_3 = in terms of T_1 and T_2

To find the values of T_3

$$(Q_{13})_{net} = (Q_{32})_{net}$$

$$\frac{T_1{}^4 - T_3{}^4}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{T_3{}^4 - T_2{}^4}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1}$$

$$\frac{T_1{}^4 - T_3{}^4}{27.33} = \frac{T_3{}^4 - T_2{}^4}{25.25}$$

$$T_1{}^4 - T_3{}^4 = \frac{27.33}{25.25}(T_3{}^4 - T_2{}^4)$$

$$T_3{}^4 = 0.48\,(T_1{}^4 + 1.08T_2{}^4)$$

Percentage reduction in heat transfer

$$= 1 - \frac{(Q_{13})}{(Q_{12})}$$

$$= 1 - \frac{\sigma(T_1{}^4 - T_3{}^4)/27.33}{\sigma\,(T_1{}^4 - T_2{}^4)/27.33}$$

$$= 1 - \frac{3.58}{27.33}\left[\frac{\left({T_1}^4 - {T_3}^4\right)}{\left({T_1}^4 - {T_2}^4\right)}\right]$$

$$= 1 - 0.131\left[\frac{{T_1}^4 - 0.48\left({T_1}^4 + 1.08\,{T_2}^4\right)}{\left({T_1}^4 - {T_2}^4\right)}\right]$$

$$= 1 - 0.131\left[\frac{0.52\left({T_1}^4 - {T_2}^4\right)}{\left({T_1}^4 - {T_2}^4\right)}\right]$$

$$= 1 - 0.131(0.52)$$

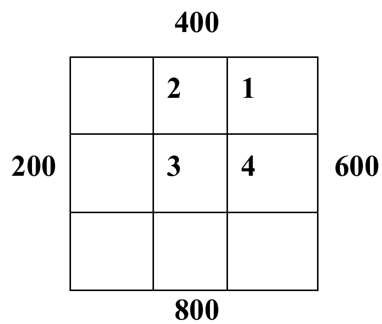
$$= 0.932$$

$$= 93.2\%$$

Unit - IV

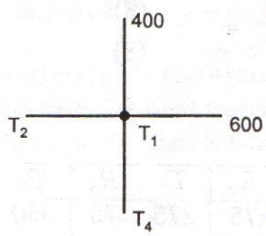
1. **Consider a two dimensional steady state heat conduction in a square region of side 'L' subject to the boundary conditions shown in the figure**

Calculate T_1, T, T_3 and T_4 considering $\Delta x = \Delta y = L/3$. Calculate the heat transfer rate through the boundary surface at x= L per 1m length perpendicular to the plane of figure for L=0.1m, k=20W/mK.



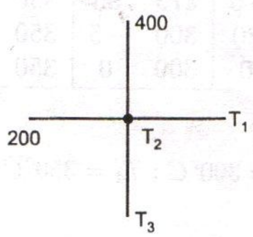
Interior nodal Temperature

i)



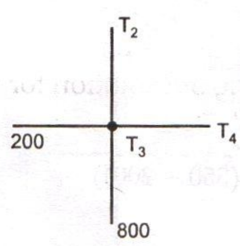
$$\therefore\ 400 + 600 + T_2 + T_4 - 4T_1 = 0 \quad ...... \quad (1)$$

ii)



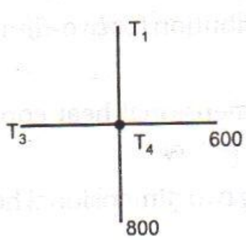
$$\therefore\ 400 + 200 + T_1 + T_3 - 4T_2 = 0 \quad ...... \quad (2)$$

iii)



$$\therefore\ 200 + 800 + T_2 + T_4 - 4T_3 = 0 \quad ...... \quad (3)$$

iv)



$$\therefore\ 800 + 600 + T_3 + T_1 - 4T_4 = 0 \quad ...... \quad (4)$$

Solution

Rearrange the questions and apply Gauss-seidel Iteration method;

$\therefore \quad 1000 + T_2 + T_4 - 4T_1 = 0$

$600 + T_3 + T_1 - 4T_2 = 0$

$1000 + T_2 + T_4 - 4T_3 = 0$

$1400 + T_1 + T_3 - 4T_3 = 0$

No. of iteration (n)	T_1	T_2	T_3	T_4
0 (assumed value)	500	300	500	700
1	500	400	525	606.25
2	501.56	406.64	503.22	601.95
3	501.95	401.29	500.81	600.69
4	500.49	400.33	500.26	600.19
5	500.13	400.09	500.07	600.05

The fourth and fifth iteration have approximately equal values

$\therefore\ T_1$=500.13°C; T_2=400.09°C; T_3=500.07°C; t_4 = 600.05°C

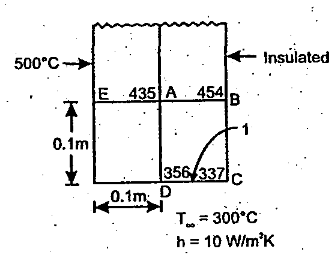
To find heat transfer rate at x=L

$$Q = k\Delta x \frac{dT}{dy} \qquad [Here \Delta y = 1]$$

$$= 20 \times 0.03333\left(\frac{(500-800)+(600.05-800)}{0.03333}\right)$$

Q = -10,000 W

2. The figure shows the temperature in a part of a solid and the boundary conditions. Estimate the thermal conductivity of the material and also find the heat flow over surface 1.



Solution:

To find heat flow from surface 1 (mode of heat transfer is convection)

$$Q = hA(\Delta T)$$

or $$Q = hA\sum(\Delta T),$$ Hear

$$A = \Delta x.\Delta y$$

(Vertical heat flow i.e heat flow from bottom face $\therefore$ unit thickness $\Delta y = 1$)

$$Q = h\Delta x\left[(T_C - T_\infty) + (T_D - T_\infty) + \frac{1}{2}(500 - 300)\right]$$

Q = 193W

$\therefore$ We know that, heat transfer is same for the material

$\therefore$ Q = kA(ΔT)

$$Q = k\Delta x[(T_A - T_D) + (T_B - T_C) + (T_E - T_F)]$$

193 = kx0.1[(435-356)+(454-337)+(500-500)]

$$\therefore k = \frac{193}{0.1\times 196}$$

k = 9.847 W/mK

3. A small cubical furnace 50 x 50 x 50 cm on the inside ISV constructed of fire clay brick (k = 10W/mK) with a wall thickness of 10 cm. The inside furnace is maintained at 500° C. Calculate the heat loss through the wall.

Given

Size of cubical furnace 50 x 50 x 50Cm.

k_b = 1.04 W/mK

L = 10cm = 10x10^{-2}m

T_i = 500^{o}C

T_o = 50^{o}C

Find Q=?

Solution

We know that Q = kS (T_i - T_o)

Cubic furnace, having 6 wall sections, 8 corners and 12 edges.

$\therefore$ Conduction shape factor for (s) wall = $\frac{A}{L} = \frac{0.5\times 0.5}{0.1}$ =2.5 m

Conduction shape factor for corner = 0.15L = 0.l5.X 0.1 = 0.015m

Conduction shape factor for edges = 0.54 D= 0.54 x 0.5 = 0.27 m

$\therefore$ Total conduction shape factor (s) = (6×2.5)+(8x0.015)+(12x0.27)

S = 18.36 m

$\therefore$ Q = kSΔT

= 1.04x18.36(500-50)

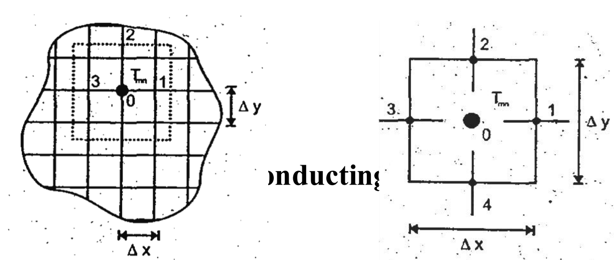
$$Q = 8.592 \text{ W}$$

4. What is meant relaxation method? Explain in detail.

- It may also be solved by "Gauss-seidel Iteration" method (For large node)
- In this method, a combined volume of the system is divided into number of sub-volumes.
- Each sub volume has a temperature distribution at its centre.
- Each sub volume has heat conducting rod. The center of each sub- volume having temperature distribution is called "nodes".

Various Steps involved in Relaxation Process

1. Subdivide the system into a number of small sub volumes and assign a reference number to each.
2. Assume values of temperatures at various nodes.
3. Using the assumed temperatures, calculate the residuals at each node.
4. Relax the largest residual to zero by changing the corresponding nodal temperature by an appropriate amount.
5. Change the residuals of the surrounding nodes to correspond with the temperature change in step (4).
6. Continue to relax residuals until all are equal to zero



$$\boxed{Q_{1\text{-}0} + Q_{2\text{-}0} + Q_{3\text{-}0} + Q_{4\text{-}0} = 0}$$

$$\frac{k.\Delta y(T_1 - T_0)}{\Delta x} + \frac{k.\Delta x(T_2 - T_0)}{\Delta y} + \frac{k.\Delta y(T_3 - T_0)}{\Delta x} + \frac{k.\Delta y(T_4 - T_0)}{\Delta y} = 0$$

$$\frac{\Delta y}{\Delta x}\left(T_1 + T_3 - 2T_0\right) + \frac{\Delta x}{\Delta y}\left(T_2 + T_4 - 2T_0\right) = 0 \qquad \text{If } \Delta X \neq \Delta Y$$

Here $\Delta x = \Delta y$

$$\therefore T_1 + T_3 + T_2 + T_4 - 4T_0 = 0$$

To find the temperature at an interior node T_0 (or) T_{mn} is

$$\boxed{T_1 + T_2 + T_3 + T_4 - 4T_0 = 0}$$

$$T_{mn} = \frac{T_{m+1,n} + T_{m,n+1} + T_{m-1,n} + T_{m,n-1}}{4}$$

5. A square plate of side L is fully insulated along the surfaces. The temperature maintained at the edges are given as:

T (x, 0) = 0

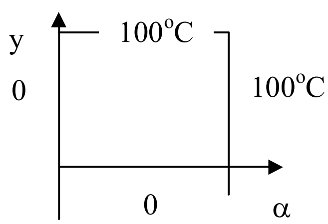
T (0, y) = 0

T (x, L) = 100°C

and T(L,y) = 100°C

Find the expression for steady state temperature distribution.

Solution:



From HMT Data book

$$T_{m,n} = \frac{1}{4}\left[T_{m+1,n} + T_{m,n+1} + T_{m-1,n} + T_{m,n-1}\right]$$

Here

$T_{m+1,n} = 100°\text{C}$

$T_{m,n+1} = 100°\text{C}$

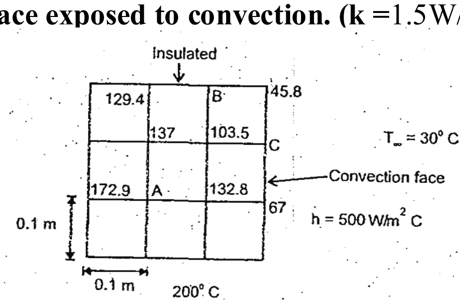
$T_{m-1,n} = 0°\text{C}$

$T_{m,n-1} = 0°\text{C}$

$$\therefore T_{m,n} = \frac{1}{4}[100 + 100 + 0 + 0]$$

$\therefore T_{m,n} = 50°\text{C}$

6. The temperature distribution and boundary condition in part of a solid is shown below; Determine the temperature at nodes marked A, B and C. Also determine the heat convected over surface exposed to convection. (k =1.5W/mK).



Solution

1. Node A is an interior node

,To find the temperature at node A

$$T_A = \frac{T_m + 1, n + T_{m-1,n} + T_{m,n+1} + T_{m,n-1}}{4}$$

$$= \frac{132.8 + 172.9 + 137 + 200}{4}$$

$T_A = 160.68^oC$

2. To find temperature at node B (it is at the insulated boundary)

$$\therefore T_B = \frac{T_{m,} n + 1 + T_{m,n-1} + 2T_{m-1,n}}{4}$$

(Refer HMT data book)

$$= \frac{129.4 + 45.8 + 2(103.5)}{4}$$

$T_B = 95.55^oC$

3. To find temperature at node C (It is at convection boundary)

$$\therefore T_C = \frac{\frac{h\Delta x}{k} T_\infty + \frac{1}{2}(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1})}{\frac{h\Delta x}{k} + 2}$$

(Refer HMT data book)

$$B_i = \frac{h\Delta x}{k} = \frac{500 \times 0.1}{1.5} = 33.33$$

$$\therefore \qquad T_C = \frac{33.33 \times 30 + \frac{1}{2}(2 \times 103.5 + 45.8 + 67)}{33.33 + 2}$$

$$T_C = 37.35^o C$$

4.18 Heat and Mass Transfer

4. Let the heat convected over surface exposed to convection.

$$Q_{Conu} = hA\Delta T$$

$$= h\Delta x \, \Delta y \sum (T - T_\infty)$$

$$= h\Delta y \left[(T - T_\infty) + (T_C - T_\infty) + (T - T_\infty) + \frac{1}{2}(T - T_\infty) \right]$$

(Unit thickness $\therefore \Delta x = 1$)

$$= 500 \times 1 \times 0.1 \left[(45.8 - 30) + (37.35 - 30) + (67 - 30) + \frac{1}{2}(200 - 30) \right]$$

$$Q = 7257.5W$$

UNIT-V

1. **Water flows at the rate of 65 kg/min through a double pipe counter flow heat exchanger. Water is heated from 50^o C to75^oC by an oil flowing through the tube. The specific heat of the oil is 1.780 kj/kg.K. The oil enters at 115^oC and leaves at 70^oC.the overall heat transfer co-efficient is 340 W/m2K.calcualte the following**

   1. Heat exchanger area
   2. Rate of heat transfer

Given:

Hot fluid – oil, Cold fluid – water

(T_1 , T_2) (t_1 , t_2)

Mass flow rate of water (cold fluid), m_c = 65 kg/min

= 65/60 kg/s

m_c = 1.08 kg/s

Entry temperature of water, $t_1 = 50^o$ C

Exit temperature of water, $t_2 = 75^o$ C

Specific heat of oil (Hot fluid), C_{ph} = 1.780 KJ/kg K

= 1.780 x 10^3 J/kg K

Entry temperature of oil, $T_1 = 115^o$ C

Exit temperature of water, $T_2 = 70^o$ C

Overall heat transfer co-efficient, U = 340 w/m^2 K

To find:

1. Heat exchanger area, (A)
2. Rate of heat transfer, (Q)

Solution:

We know that,

Heat transfer, $Q = m_c\, c_{pc}\, (t_2 – t_1)$ (or) $m_h\, c_{ph}\, (T_1 - T_2)$

$Q = m_c\, C_{pc}\, (t_2 – t_1)$

$Q = 1.08 \times 4186 \times (75 – 50)$

[Specific heat of water, c_{pc} = 4186 J/kg K]

$Q = 113 \times 10^3$ W

We know that,

Heat transfer, $Q = U \times A\, (\Delta T)_m$ …….. (1)

[From HMT data book page no:152(sixth edition)]

Where

ΔT_m – Logarithmic Mean Temperature Difference. (LMTD)

For counter flow,

$$\Delta T_{lm} = \frac{[(T_1 - t_2) - (T_2 - t_1)}{ln\left[\frac{T_1 - t_2}{T_2 - t_1}\right]}$$

$$\Delta T_{lm} = \mathbf{28.8^oC}$$

Substitute $(\Delta T)_{lm}$, Q and U values in Equn (1)

(1) $Q = UA\ (\Delta T)_{lm}$

$113 \times 10^3 = 340 \times A \times 28.8$

$A = 11.54\ m^2$

2. A parallel flow heat exchanger is used to cool 4.2 kg/min of hot liquid of specific heat 3.5 kJ/kg K at 130° C. A cooling water of specific heat 4.18 kJ/kg K is used for cooling purpose of a temperature of 15° C. The mass flow rate of cooling water is 17 kg/min. calculate the following.

1. Outlet temperature of liquid
2. Outlet temperature of water
3. Effectiveness of heat exchanger

Take

Overall heat transfer co-efficient is 1100 W/m^2 K.

Heat exchanger area is $0.30m^2$

Given:

Mass flow rate of hot liquid, $m_h = 4.2$ kg/min

$m_h = 0.07$ kg/s

Specific heat of hot liquid, $c_{ph} = 3.5$ kJ/kg K

$c_{ph} = 3.5 \times 10^3$ J/kg K

Inlet temperature of hot liquid, $T_1 = 130^0$ C

Specific heat of hot water, $C_{pc} = 4.18$ kJ/kg K

$C_{pc} = 4.18 \times 10^3$ J/kg K

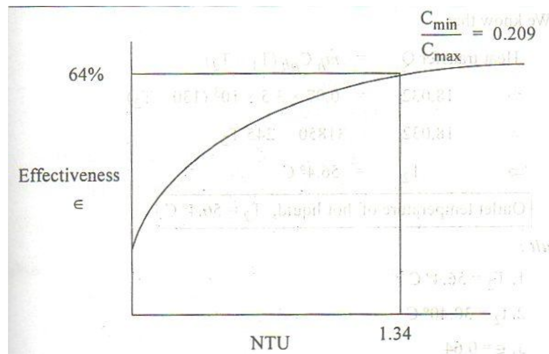
Inlet temperature of hot water, $\mathbf{t_1} = 15^0$ C

Mass flow rate of cooling water, $m_c = 17$ kg/min

$$\mathbf{m_c = 0.28 \text{ kg/s}}$$

Overall heat transfer co – efficient, U = 1100 w/m^2 K

Area, A = 0.03 m^2



To find :

1. Outlet temperature of liquid, (T_2)
2. Outlet temperature of water, (t_2)
3. Effectiveness of heat exchanger, (ε)

Solution :

Capacity rate of hot liquid, $C_h = m_h \times C_{ph}$

$= 0.07 \times 3.5 \times 10^3$

$$\mathbf{C_h = 245 \text{ W/K}}\text{ .... } \mathbf{(1)}$$

Capacity rate of water, $C_c = m_c \times C_{pc}$

$= 0.28 \times 4.18 \times 10^3$

$$\mathbf{C_c = 1170.4 \text{ W/K}}\text{ .... } \mathbf{(2)}$$

From (1) and (2),

$C_{min} = 245$ W/K

$C_{max} = 1170.4$ W/K

$$=> \frac{C_{min}}{C_{max}} = \frac{245}{1170.4} = 0.209$$

$$\mathbf{\frac{C_{min}}{C_{max}} = 0.209} \text{ .... } \mathbf{(3)}$$

Number of transfer units, NTU = $\frac{UA}{C_{min}}$

[From *HMT data book page no. 152]*

$$=> \quad NTU = \frac{1100 \times 0.30}{245}$$

$$\mathbf{NTU = 1.34} \text{ .... } \mathbf{(4)}$$

To find effectiveness ε, refer HMT data book page no 163

(Parallel flow heat exchanger)

From graph,

$X_{axis} \rightarrow$ NTU = 1.34

Curve $\rightarrow \frac{C_{min}}{C_{max}} = 0.209$

Corresponding Y_{axis} value is 64 %

i.e., **ε = 0.64**

from HMT data Book

$$\in = \frac{m_h cp_h (T_1 - T_2)}{C_{min}(T_1 - t_1)}$$

$$0.64 = \frac{130 - T_2}{130 - 15}$$

$T_2 = 56.4\ ^oC$

To find t_2

$m_h\ cp_h(T_1\text{-}T_2) = m_c Cp_c\ (t_2\text{-}t_1)$

$0.07 \times 3.5 \times 10^3\ (130\text{-}56.4) = 0.28 \times 4186\ (t_2\text{-}15)$

$t_2 = 30.4^oC$

Maximum possible heat transfer

$Q_{max} = C_{min}(T_1 - t_1)$

$= 245\ (130 - 15)$

$\mathbf{Q_{max} = 28.175\ W}$

Actual heat transfer rate

$Q = \varepsilon \text{ x } Q_{max}$

$= 0.64 \text{ x } 28.175$

$\mathbf{Q = 18.032\ W}$

We know that,

Heat transfer, $Q = m_c\ C_{pc}(t_2 - t_1)$

=> $18.032 = 0.28 \text{ x } 4.18 \text{ x } 10^3\ (t_2 - 15)$

=> $18.032 = 1170.4\ t_2 - 17556$

=> $t_2 = 30.40^oC$

Outlet temperature of cold water, $t_2 = 30.40^oC$

We know that,

Heat transfer, $Q = m_h\ C_{ph}(T_1 - T_2)$

=> $18.032 = 0.07 \text{ x } 3.5 \text{ x } 10^3\ (130 - T_2)$

=> $18.032 = 31850 - 245\ T_2$

=> $T_2 = 56.4^oC$

Outlet temperature of hot liguid, $T_2 = 56.4^oC$

3.Hot chemical products (C_{ph} = 2.5 kJ/kg K) at 600° C and at a flow rate of 30 kg/s are used to heat cold chemical products (C_p = 4.2 kJ/kg K) at 200° C and at a flow rate 20 kg/s in a parallel flow heat exchanger. The total heat transfer is 50 m^2 and the overall heat transfer coefficient may be taken as 1500 W/m^2 K. calculate the outlet temperatures of the hot and cold chemical products.

Given: Parallel flow heat exchanger

$T_{h1} = 600^{o}$ C ; m_h = 30 kg/s

C_{ph} = 2.5 kJ/kg K

$T_{c1} = 100^{o}C$; m_c 28 kg/s

C_{pc} = 4.2kJ/kg K

A = 50m^2

U = 1500 W/m^2K

Find:

(i) T_{h2} (ii) T_{c2} ?

Solution

The heat capacities of the two fluids

$C_h = m_h c_{ph}$ = 30 x 2.5 =75 kW/K

$C_c = m_c c_{pc}$ = 28 x 4.2 = 117.6 kW/K

The ratio $\frac{C_{min}}{C_{max}} = \frac{75}{117.6} = 0.64$

$$NTU = \frac{UA}{Cmin} = \frac{1500 \, x \, 50}{75 \, x \, 10^3} = 1.0$$

For a parallel flow heat exchanger, the effectiveness from Fig. 13.15 corresponding to $\frac{C_{min}}{C_{max}}$ and NTU

$\in$ = **0.48**

We know that

$$\in = \frac{Actual\ heat\ transfer}{Max.possible\ heat\ transfer}$$

$$= \frac{m_h C_{ph}(T_{h1} - T_{h2})}{C_{min}(T_{h1} - T_{c1})}$$

$$\in = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c1})}$$

$$0.48 = \frac{600 - T_{h2}}{600 - 100}$$

T_{h2} = 360°C

We know that

Heat lost by the hot product = Heat gained by the cold product

$$m_h c_{ph} (T_{h1} - T_{h2}) = m_c c_{ph} (T_{c2} - T_{c1})$$

$$75(600 - 360) = 117.6\ (T_{c2} - 100)$$

$$\boldsymbol{T_{c2} = 253.06^o\ C}$$

4. Estimate the diffusion rate of water from the bottom of a tube of 10mm diameter and 15cm long into dry air 25°C. Take the diffusion coefficient of water through air as 0.235 x 10^{-4}m²/s

Given:

$D = 0.255 \times 10^{-4} m^2/s$

Area (A) $= \frac{\pi}{4}\ d^2 = \frac{\pi}{4}\ (0.01)^2 = 7.85\ x\ 10^{-5}\ m^2$

R_o = 8314 J/kg – mole K

T = 25 + 273 = 298 K

M_w = molecular weight of water = 18

P = Total pressure = 1.01325×10^5 N/m²

$X_2 - X_1$ = 0.15m

P_{w1} = partial pressure at 25° C = 0.03166×10^5 N/m²

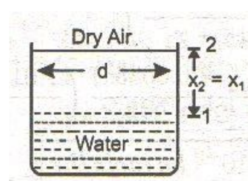
$P_{w2} = 0$

Find:

Diffusion rate of water (or) Mass transfer rate of water.

Solution

We know that



Molar rate of water (M_a)

$$M_a = \frac{DA}{R_o T} \cdot \frac{P}{x_2 - x_1}\ \text{In}\left(\frac{P_{a2}}{P_{a1}}\right)$$

$$= \frac{0.255 \text{ x } 10-4 \text{ x } 7.85 \text{ x } 10-5 \text{ x } 1.01325 \text{ x } 105}{8314 \text{ x } 298 \text{ x}0.15} \text{ x } \left(\frac{1.01325-0}{1.01325-0.03166}\right)$$

Here $P_{a2} = P - P_{w2}$, $P_{a1} = P - P_{w1}$

M_a = 1.72 x 10^{-11} kg-mole/s

Mass transfer rate of water
(or)
Diffusion rate of water

= Molar rate of water X molecular weight of steam

$M_w = 1.72 \times 10^{-11} \times 18$

Diffusion rate of water (M_w) = 3.1 x 10^{-10} kg/s

5. A vessel contains a binary mixture of O_2 and N_2 with partial pressure in the ratio of 0.21 and 0.79 at 15°C. The total pressure of the mixture is 1.1 bar. Calculate the following

1. Molar concentration
2. Mass densities
3. Mass fractions
4. Molar fractions.

Given:

T = 15 + 273 = 288 K

P = 1.1bar = 1.1 x 10^5 N/m^2

P_{o_2} = 0.21 bar

P_{N_2} = 0.21 bar

Solution

1. To find Molar concentration (C_{o_2} and C_{o_2})

$$C_{O_2} = \frac{P_{O_2}}{R_o T} = \frac{0.21 \times 1.1 \times 10^{5}}{8314 \times 288}$$

$$\mathbf{C_{O_2} = 0.00965\ kg\ mole/m^3}$$

$$C_{N_2} = \frac{P_{N_2}}{R_o T} = \frac{0.79 \times 1.1 \times 10^{4}}{8314 \times 288}$$

$$\mathbf{C_{N_2} = 0.0363\ kg\ mole/m^3}$$

2. To find mass densities (p_{o_2} and p_{N_2})

$$\boldsymbol{P} = \mathbf{MC}$$

Where, M: Molecular weight

$$P_{o_2} = M_{o_2} \times C_{o_2} = 32 \times 0.00965$$

$$\mathbf{P_{o_2} = 0.309\ kg/m^3}$$

$$P_{N_2} = M_{N_2} \times C_{N_2} = 28 \times 0.0363$$

$$\mathbf{P_{N_2} = 1.016\ kg/m^3}$$

3. To find mass fractions (M_{o_2} and M_{N_2})

We know that

$$\rho = \rho_{o_2} + \rho_{N_2} = 0.309 + 1.016$$

$$\boldsymbol{\rho = 1.375\ kg/m^3}$$

$$M_{o_2} = \frac{\rho_{o_2}}{\rho} = \frac{0.309}{1.325}$$

$$\boldsymbol{M_{o_2} = 0.233}$$

$$M_{N_2} = \frac{\rho_{N2}}{\rho} = \frac{1.016}{1.325}$$

$$\boldsymbol{M_{N_2} = 0.767}$$

4. To find molar fraction (n_{o_2} and n_{N_2})

   We know that

$$C = C_{o_2} + C_{N_2} = 0.00965 + 0.0363$$

$$C = \boldsymbol{1.375\ kg\ mole/m^3}$$

$$n_{o_2} = \frac{C_{o_2}}{C} = \frac{0.00965}{0.046}$$

$$\boldsymbol{n_{o_2} = 0.21}$$

$$n_{N_2} = \frac{C_{N_2}}{C} = \frac{0.0363}{0.046}$$

$$\boldsymbol{n_{N_2} = 0.79}$$

6. A counter flow heat exchanger is employed to cool 0.55 kg/s (C_p = 2.45 kj/kg°C) of oil from 115°C to 40°C by the use of water. The inlet and outlet temperature of cooling water are 15°C and 75°C respectively. The overall heat transfer coefficient is expected to be 1450 W/m^2°C.

Using NTU method, calculate the following:

(i) The mass flow rate of water.

(ii) The effectiveness of heat exchanger.

(iii) The surface area required.

Given:

Counter flow HE

M_h = 0.55 kg/s

C_{p_h} = 2.45kj/kg°C

T_1 = 115°C

T_2 = 40°C

t_1 = 15°C

t_2 = 75°C

U = 1450 W/m^2°C

To find:

1.The mass flow rate of water. (m_c)

2.The effectiveness of heat exchanger. ($\in$)

3.The surface area required.(A)

Solution:

For $\in -NTU$ method from HMT date book

$$\mathbf{Q} = \in \mathbf{C_{min}\,(T_1 - t_1)}$$

To find m_c

Use energy balance equation.

Heat lost by hot fluid = Heat gained by cold fluid

$$m_h C_{p_h}(T_1 - T_2) = m_c C_{p_c}(t_2 - t_1)$$

0.55 x 2450 (115 - 40) = m_c x 4186 (75 - 15)

m_c = 0.40kg/s

Heat capacity rate of hot fluid = $C_h = m_h - C_{p_h}$

= 0.55 x 2.45

C_h = 1.35 kw/K

Heat capacity rate of cold fluid = $C_c = m_c - C_{p_c}$

= 0.40 x 4.186

C_c = 1.67kw/K

$C_h < C_c$

$C_h = C_{min}$

$$\in = \frac{m_h C_{p_h}(T_1 - T_2)}{C_{min}\ (T_1 - T_2)}$$

$$= \frac{115-40}{115-15}$$

$\in = 0.75 = 75\%$

Q = 0.75 x 1350 (115 – 15)

Q = 101.250W

$Q = UA\ (\Delta T)_{lm}$

$A = Q/U(\Delta T)_{lm}$

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{ln\left[\frac{(T_1 - t_2)}{(T_2 - t_1)}\right]}$$

$$= \frac{(115-75)-(40-15)}{ln\left[\frac{115-75}{40-15}\right]}$$

$(\Delta T)_{lm} = 31.9^oC$

$$A = \frac{101.250}{1450\ x\ 31.9}$$

A = 2.19 m^2

7. A pan of 40 mm deep, is filled with water to a level of 20 mm and is exposed to dry air at 30^0 C. Calculate the time required for all the water to evaporate. Take, mass diffusivity is 0.25X10 $^{-4}$ m^2/s.

Given:

Deep, $(x_2 - x_1)$ = 40 – 20 = 20 mm = 0.020 m

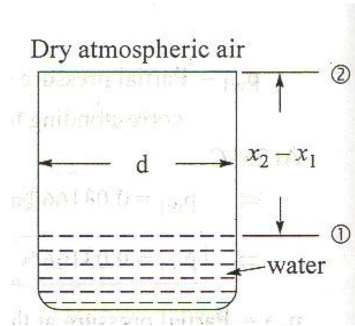
Temperature, T = 30^0 C + 273 = 303 K

Diffusion co- efficient , D_{ab} =0.25X10 $^{-4}$ m^2/s.

To find:

Time required for all the water to evaporate, t.

Solution:



We know that, for isothermal evaporation

Molar flux, $\frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{p}{(x_2-x_1)} \times \ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]$ …………(1)

Where,

G – Universal gas constant = 8314 j/kg – mole-K

P – Total pressure = 1 atm = 1.013 bar = $1.013 \text{ X } 10^5$ N/m^2

p_{w1} - Partial pressure at the bottom of the pan Corresponding to saturation temperature 30°C

At 30°C

$\Rightarrow \quad p_{w1} = 0.04242 \text{ bar}$ (From steal table page no.2)

$\Rightarrow \quad p_{w1} = 0.4242\times10^5 \text{ N/m}^2$

P_{w2} – partial pressure at the top of the pan, which is zero.

$\Rightarrow \quad P_{w2} = 0$

$$(1) \Rightarrow \quad \frac{m_a}{A} = \frac{0.25\times10^{-4}}{8314\times303}\times\frac{1.013\times10^5}{0.020}\times\left[\frac{1.013\times10^5-0}{1.013\times10^5-0.04242\times10^5}\right]$$

$$\frac{m_a}{A} = 2.15\times10^{-6}\frac{kg-mole}{s}$$

For unit Area, A = 1m²

Molar rate of water, m_a=$2.15 \times 10^{-6} \frac{kg-mole}{sm^2}$

We know that,

Mass Rate of water vapour = Molar Rate of water vapour × Molecular weight of steam

$= 2.15\times10^{-6}\times18.016$

Molar rate of water vapour $= 3.87\times10^{-5}$ kg/s-m²

The total amount of water to be evaporated per m² area

$= (0.20\times1) \times1000$

$=20$ kg/m² Area

$$\text{Time required, } t = \frac{20}{\text{Mass rate of water vapour}}$$

$$= \frac{20}{3.87\times10^{3}s}$$

Result :

Time required for all the water to evaporate, $t=516.79\times10^{3}S$

8. A heat exchanger is to be designed to condense an organic vapour at a rate of 500 kg/min. Which is available at its saturation temperature of 355 K. Cooling water at 286 K is available at a flow rate of 60 kg/s. The overall heat transfer coefficient is 475 W/m^{2}C Latent heat of condensation of the organic vapour is 600 kJ/kg. Calculate

1. The number of tubes required, if tubes of 25 mm otuer diameter, 2mm thick and 4.87m long are available, and
2. The number of tube passes, if cooling water velocity (tube side) should not exceed 2m/s.

Given:

d_o = 25 mm = 0.025

d_i = 25-(2×2)= 21 mm = 0.21 m

L = 4.87 m

V = 2 m/s

T_{c1} = 286-273 = 13^{o}C

T_{sat} = T_{h1} = T_{h2} = 355-273 = 82^{o}C

U = 475 /m^2 K

h_{fg} = 600 kj/kg

m_h = $\frac{500}{60}$ = 8.33 kg/s

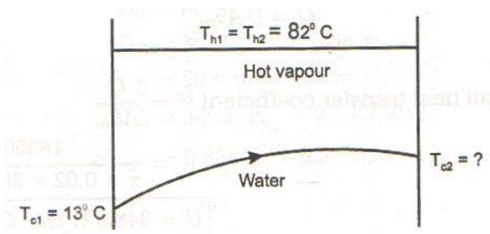
m_c = 60 kg/s

Find

(i) Number of tubes (N)

(ii) Number of tube passes (P)

Solution



$Q = UA\theta_m=U(\pi d_oLN)\theta m$

$Q = m_hh_{fg} = m_cC_{pc}(T_{c2}-T_{c1})$

i.e. Heat lost by vapour = heat gained by ater

$Q = 8.33 \times 600 \times 10^3$

$Q = m_c c_{pv} (T_{c2} - T_{c1})$

$8.33 \times 600 \times 10^3 = 60 \times 4.18\,(T_{c2} - 13)$

$T_{c2} = 32.9^oC$

$$\therefore \theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\dfrac{\theta_1}{\theta_2}\right)}$$

$$\theta_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left(\dfrac{(T_{h1} - T_{c1})}{(T_{h2} - T_{c2})}\right)}$$

$$= \frac{(82 - 13) - (82 - 32.9)}{\ln\left(\dfrac{(82 - 13)}{(82 - 32.9)}\right)}$$

$$\theta_m = 58.5^o C$$

Heat transfer rate is given by

$$Q = m_h h_{fg} = UA\theta_m$$

$8.33 \times 600 \times 10^3 = 475 \times (\pi \times 0.025 \times 4.87 \times N \times 58.5)$

N= 470 Tubes

To find N. of tube passes (P)

$N = P \times N_p$

Where

N	:	No. of tubes
P	:	No. of tube passes
N_p	:	No. of tubes in each pass

i.e. The cold water flow passing through each pass.

$$m_c = AV_p N_p$$

$$60 = \frac{\pi}{4} di^2 V_\rho \times N_p$$

$$60 = \frac{\pi}{4}(0.021)^2 \times 2 \times 1000 \times N_p$$

$N_p = 95.5$

We know that

$N = P \times N_p$

$\therefore$ No. of passes (P) $= \frac{N}{N_p}$

$$= \frac{470}{95.5} = 4.91$$

P = 5

$\therefore$ **Number of passes (P) = 5**

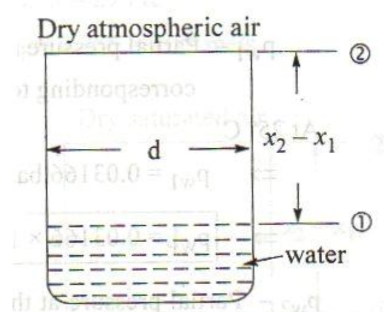
9. An Open pan 20 cm in diameter and 8 cm deep contains water at 25°C and is exposed to dry atmospheric air. If the rate of diffusion of water vapour is 8.54×10^{-4} kg/h, estimate the diffusion co-efficient of water in air.

Given

Diameter d	=	20 cm	=	0.20 m
Length (x_2-x_1) =		8cm	=	0.08 m
Temperature ,T=		25°C+273	=	298 K
Diffusion rate (or)				
Mass rate of water vapour			=	8.54×10^{-4} kg/h
			=	$\frac{8.54\times10^{-4}\,kg}{3600\,s}$
			=	2.37×10^{-7} kg/s

To find

Diffusion co-efficient, D_{ab}



Solution

We know that

Molar rate of water vapour

$$\frac{m_a}{A} = \frac{D_{ab}}{GT}\frac{p}{(x_2-x_1)}\times\ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]$$

$$\Rightarrow m_a = \frac{D_{ab}}{GT}\frac{p}{(x_2-x_1)}\times\ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]$$

We know that,

Mass rate of water vapour = Molar rate of water vapour + Molecular weight of steam

$$2.37 \text{ x } 10^{-7} = \frac{D_{ab}}{GT}\frac{p}{(x_2-x_1)}\times\ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]\times 18.016 \quad \text{......... (1)}$$

where,

Area, A $= \frac{\pi}{4}d^2$

$= \quad \frac{\pi}{4}(0.20)^2$

A $=$ 0.0314 m^2

G - Universal gas constant = 8314 $\frac{J}{kg - mole - K}$

p - Total pressure = 1 atm = 1.013 bar

= 1.013×10^5 N/m^2

p_{wl} = Partial pressure at the bottom of the test tube corresponding to saturation temperature 25°C

At 25°C

$\Rightarrow p_{wl}$ = 0.03166 bar [From (R.S. Khurami) Steam table, Page no.2]

$\Rightarrow p_{wl}$ = 0.03166×10^5 N/m^2

p_{w2} - Partial pressure at the top of the pan. Here, air is dry and there is no water vapour. So, pw2 – 0.

$\Rightarrow$ $p_{w2} = 0$

(1) $2.37 \times 10^{-7} =$

$$\frac{D_{ab} \times 0.0314}{8314 \times 298} \times \frac{1.013 \times 10^5}{0.08} \times in\left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.03166 \times 10^5}\right] \times 18.016$$

D_{ab} = 2.58×10^{-5} m^2/s

Result:

Diffusion co-efficient, $D_{ab} = 2.58 \times 10^{-5}$ m^2/s.

10. A counter flow double pipe heat exchanger using super heated steam is used to heat water at the rate of 10500 kg/hr. The steam enters the heat exchanger at 180°C and leaves at 130°C. The inlet and exit temperature of water are 30°C and 80°C respectively. If the overall heat transfer coefficient from steam to water is 814 W/m² K, calculate the heat transfer area. What would be the increase in area if the fluid flow were parallel?

Given

Counter flow heat exchanger

$$\dot{m}_w = \dot{m}_c = \frac{10500}{3600} = 2.917\, kg/s$$

T_1 = 180°C t_1=30°C

T_2 = 130°C t_2 = 80°C

U = 814 W/m^2 K

Find

(i) Area of heat transfer (A)

(ii) Increase in area

Solution

(i) When the flow is counter:

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)}$$

$$\theta_1 = T_1 - t_2 = 180 - 80 = 100^o C$$

$$\theta_2 = T_2 - t_1 = 130 - 30 = 100^o C$$

LMTD = 0 ^{o}C

If LMTD = 0 ^{o}C use AMTD

So, AMTD $= \dfrac{\theta_1 + \theta_2}{2}$ [AMTD: Arithmetic mean temperature difference]

AMTD $= \dfrac{100 + 100}{2}$

AMTD = 100^{o}C

θ_m = 100^{o}C

Here ΔT_{lm} = AMTD

$\therefore$ To find heat transfer rate

$$\boxed{Q = U\,A\,\Delta T_{lm}}$$

$$\boxed{Q = \dot{m}_c c_{pc}(t_2 - t_1)}$$

Q = 2.917×4.187 ×10^3 (80-90)

2.917×4.187 ×10^3 ×50 = 814 × A ×100

$$\boxed{A = 7.5\ m^2}$$

ii) When the flow is parallel

$$\Delta T_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln[((T_1 - t_1)/(T_2 - t_2))]}$$

$$= \frac{(180 - 30) - (130 - 80)}{\ln[(180 - 30)/(130 - 80)]}$$

$$= \frac{150-50}{\ln\left[150/50\right]} = 91\,^{o}C$$

$Q \quad = U\ A\ \Delta T_{lm}$

or $2.917 \times (4.187\times 10^3) \times (80-30) = 814 \times A \times 91$

$A \quad = 8.24\, m^2$

$\therefore$ Increase in Area = $\frac{8.24-7.5}{7.5} = 0.0987$ *or* 9.87%